MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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F. W. DYSON, Esq., M.A., F.R.S., PRESIDENT, in the Chair.

The Rev. William Aitken Haslam, M.A., 19 Dalebury Road, Wandsworth Common, S.W.;
Evan Lewis Jones, B.Sc., 12 Ladysmith, Troedyrhin, Merthyr-Tydfil, South Wales; and
Henry Morris, B.Sc. (Lond.), Technical and County Secondary School, Beckenham, and 44 Belmont Road, Beckenham, Kent,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Ernest Herbert Beattie, The Observatory, Orlando Avenue, West Point, Mosman, Sydney, New South Wales (proposed by G. D. Hirst);
Hector Macpherson jun., Alpine Villa, Currie, Midlothian, Scotland (proposed by F. W. Dyson); and
Frederick Slocum, Yerkes Observatory, Williams Bay, Wisconsin, U.S.A. (proposed by S. W. Burnham).

Seventy-nine presents were announced as having been received since the last meeting, including, amongst others:—
Astronomical and Astrophysical Society of America, Publica-

tions, vol. 1, presented by the Society; Oskar Backlund, La Comète d'Encke, 1891–1908, fasc. 3, presented by the Author; K. Pokrowski, Der Schweif des Kometen 1910 *a*, presented by the Author.

Determination of the Moon's Parallax from Meridian Observations of the crater Mösting A at the Royal Observatories of Greenwich and the Cape of Good Hope in the Years 1906–1910.

(*Communicated by the Astronomer Royal and His Majesty's Astronomer at the Cape.*)

In the course of the year 1904 it was arranged between Sir William Christie and Sir David Gill that, commencing with January 1905, the lunar crater Mösting A should be observed on the meridian at both observatories whenever practicable. The object was twofold:—both to give improved observations of Right Ascension (avoiding the discontinuity at full moon from the change of illuminated limb from the first to the second), and also to give an improved observational value of the parallax, owing to the smaller liability to error that there is in placing a wire so as to bisect a small bright crater, than in placing it so as to touch the Moon's limb. In addition to complications of irradiation, and the well-known large personalities that exist in limb observations, there is the fact that the Moon's limb is frequently mountainous, and there is room for a good deal of judgment on the part of the observer in placing a wire tangent to the mean limb. It was not at first anticipated that so many years would be required to give sufficient observational material; but in fact the number of days in each year when the crater was observed at both stations was only about 20; and as a few observations have had to be rejected for various reasons, it has taken six years to obtain 100, which was fixed on as the minimum for a satisfactory determination.

The programme of observations included the observation of the Moon Culminating Stars in North Polar Distance, to enable the observations of the crater at both observatories to be reduced to the same system, and to eliminate all uncertainties of flexure, refraction, zenith point, division error, etc. As, however, there were many nights on which the full number of culminators was not observed, other fundamental stars have been utilised (in the case of Greenwich), selecting them whenever possible so as to make their mean N.P.D. the same as that of the Moon.

It was clearly essential, where the same stars were not employed at the two stations, to use a definite system of assumed star-places. These have accordingly been derived from Boss's Provisional General Catalogue for all stars contained in that work, and for other stars the mean has been taken of the following

Zodiacal Catalogues for 1900—Edinburgh (1898 to 1903), the Cape (1900 to 1904), and the Lick Observatory (1901 to 1904). Equal weight has been given to the three, except south of Declination $-14°$, when half weight is given to Edinburgh, in consequence of the small altitude. Only about 3 stars were left unprovided for, and their places were taken from the Washington Zodiacal Catalogue (*American Ephemeris Papers*, vol. viii.) half weight being given to them, owing to the less recent date of the observations.

With star-places determined as above, an Equator-Point was adopted for each instrument, and the observed N.P.D. of the crater corrected accordingly.

The parallaxes were calculated on two assumptions of the Earth's ellipticity, viz. $\frac{a-b}{a} = 1/293{\cdot}5_{(1)}$ and $= 1/300_{(2)}$.

Assuming the following coordinates for the different instruments—

	Latitude.	Altitude.
	° ′ ″	
Greenwich Transit Circle	+51 28 38·25	154 feet
,, Altazimuth	+51 28 36·58	176 ,,
Cape Transit Circle	−33 56 3·58	42 ,,
,, Reversible T.C.	−33 56 2·53	38 ,,

—the following Geocentric coordinates were calculated on the two assumptions:—

	Geocentric Latitude.		Log ρ.		
	° ′ ″	″			
Gr. T.C.	+51 17 11·67 (1)	26·58 (2)	9·999	0996 (1)	1192 (2)
Gr. Alt.	+51 17 10·00 (1)	24·91 (2)	9·999	1000 (1)	1196 (2)
Cape T.C.	−33 45 12·31 (1)	26·43 (2)	9·999	5424 (1)	5523 (2)
Cape R.T.C.	−33 45 11·26 (1)	25·38 (2)	9·999	5424 (1)	5523 (2)

The Geocentric Zenith distances were formed and the parallaxes calculated by the formula—

$$\text{Log parallax in seconds of arc} = \text{Log sin Appt. Z.D.} + \text{Log sin } p_K + \text{Log } \rho + \text{Log } 206264''{\cdot}8 + \text{Log} \frac{\text{Arc}}{\text{Sine}} \text{ Approx. Parallax.}$$

Log sin p_K (*i.e.* the Log sine horizontal equatorial parallax of the crater Mösting A) has been interpolated with second differences from the *Berliner Jahrbuch*, and it has been assumed that the value given there is consistent with the Nautical Almanac Parallax for the Moon's Centre (*i.e.* Hansen's Parallax, with Newcomb's correction applied to the Moon's Mean Anomaly).

Examination was made whether the correction found by Dr.

Cowell to the eccentricity of the Moon's orbit would enter sensibly into the parallax. He found 22639″·48 for the argument of the Principal Elliptic Inequality as compared with Hansen's 22640″·17. The effect on the parallax in apogee and perigee is only 0″·006, and being of a periodic nature it may safely be neglected altogether. In other words, we have only one unknown in each equation of condition, viz. the correction required to Hansen's Mean Parallax.

After applying the calculated Parallaxes and obtaining the Geocentric N.P.D.s of the crater, the corrections for motion have to be applied to reduce the Cape observation to the same instant as the Greenwich one. The formula for these is—

Tabular N.P.D. of ☽ centre at Time of Greenwich Observation,
− Tabular N.P.D. of ☽ centre at Time of Cape Observation,
+ Tabular value of $\delta_{\mathfrak{D}} - \delta_{\mathfrak{K}}$ at Time of Greenwich Observation,
− Tabular value of $\delta_{\mathfrak{D}} - \delta_{\mathfrak{K}}$ at Time of Cape Observation,

the N.P.D. of ☽ centre being interpolated with second differences from the *Nautical Almanac*, and the rate of change of $\delta_{\mathfrak{D}} - \delta_{\mathfrak{K}}$ (viz. geocentric Decl. of ☽ centre − geocentric Decl. of crater) being taken from the *Berliner Jahrbuch*.

The equation of condition is then formed as follows:—

$$\left.\begin{array}{l}\text{Correction required}\\ \text{to Hansen's value}\\ \text{of Mean Parallax}\end{array}\right\} = \left\{\begin{array}{l}\text{Geocentric N.P.D.}\\ \text{from Greenwich Ob-}\\ \text{servation − Geo-}\\ \text{centric N.P.D. from}\\ \text{Cape Observation}\\ \text{corrected for Motion}\end{array}\right\} \times \frac{57'}{\text{Sum of parallaxes.}}$$

The sum of parallaxes means the sum of the calculated parallaxes of the crater for Greenwich and the Cape.

In all cases the above equation was formed on both assumptions of the Earth's ellipticity: the difference between the correction to the parallax on the two assumptions is, however, very nearly constant—

viz. correction to Hansen on 1/293·5 assumption
− correction to Hansen on 1/300 assumption $= +0''\cdot37 \pm 0''\cdot02.$

It is accordingly considered sufficient to give here the details of the discussion on the first assumption, and simply the final value on the second assumption.

In the following table the Apparent N.P.D. of the crater at the two stations has been corrected for the reduction to Boss's system or that of the Zodiacal Catalogues, but uncorrected for parallax. The instruments employed are denoted as follows; T,A in the Greenwich column denote the Transit Circle and Altazimuth respectively; T,R in the Cape column denote the Transit Circle and Reversible Transit Circle respectively. Where no letter is

put, observations were made with both instruments at that station.

To save space in printing the same degree of Apparent N.P.D. applies to both stations, the minutes being carried beyond 60.

Log sin p_K is given in the *Berliner Jahrbuch* to 5 decimals only; it is here interpolated to 7, but the last two are unreliable. The characteristic is always 8 and is not printed.

The value printed corresponds to the Greenwich observation; the application of the correction in the following column (in units of the 7th decimal) will give the value used for the Cape Observation.

The weights have been assigned as follows:—

Wt.

2 when the crater is observed with both instruments at each station.

1½ when the crater is observed with both instruments at one station.

1 when the crater is observed with one instrument at each station.

Where weights of ½ or 0 are assigned the reason is given in the notes.

In the column "observer," when both instruments were used at either station, the observer with the Transit Circle is put first. The microscopes were frequently read by a different observer from the one at the Telescope, but only the latter is given here, since personality can hardly arise in the microscope reading if the same observer read the microscopes for the crater and the stars.

The following table gives the observers:—

Greenwich.		*Cape.*	
A=H. Acton.	HF=H. H. Furner.	RC=R. W. Cheeseman.	P=R. T. Pett.
C=A. C. D. Crommelin.	J=F. Jeffries.	C=W. H. Cox.	JP=J. Power.
C=R. T. Cullen.	JS=J. Storey.	JJ=J. W. Jackson.	S=F. H. Scragg.
D=S. Daniels.	V=E. V. Vagg.	J=C. W. Jeffries.	WW=W. Whittingdale.
E=B. D. Evans.	W=W. M. Witchell.	M=H. F. Mullis.	AW=A. J. Wilkie.
E=J. E. Evans.	AW=A. Witney.	AP=J. A. J. Pead.	JW=J. C. Wood.

Date.	Observers. Gr.	Cape.	Apparent N.P.D. of Crater. Greenwich.	Cape.	Log sin p_K. Greenwich.	Corr. for Cape.	Motion of ☽ in N.P.D. Gr. – Cape.
			° ′ ″	′ ″			′ ″
1905.							
Mar. 15	AC	J, JP	70 124 55·39A	45 40·60	·2256530	−3764	+ 1 27·47
17	AC	J, M	73 147 38·25A	57 48·03	·2405130	−3502	+ 8 32·50
22	HF	AW	96 124 0·10A	29 5·40T	·2511030	+1894	+14 15·46
Apr. 14	V	M	75 121 55·88A	30 55·73T	·2373120	−3351	+10 0·42
May 15	V	JP, RC	91 116 11·76A	19 13·36	·2475400	−1245	+15 12·84
23	HF	C	105 88 6·23A	25 47·41T	·2150420	+3163	− 7 34·65
June 19	V	J	106 104 38·03A	41 2·25T	·2207430	+3016	− 6 28·77
July 13	RC	M	107 98 1·99T	20 27·69R	·2341673	+1417	+ 5 42·49
15	E	P	108 110 55·76T	42 51·61T	·2269615	+2187	− 1 44·74
20	W	JW	95 72 6·78T	11 34·06T	·2043176	+1689	−12 4·51
Aug. 11	SD	AW	108 115 47·27T	46 41·88T	·2247785	+1952	− 0 20·67
Sept. 8	V	JP	108 93 18·02A	27 3·54T	·2204090	+2367	− 2 40·43
12	HF	RC	98 73 2·61A	12 49·31T	·2049680	+1544	−11 26·28
Oct. 10	HF	AW	95 99 39·92A	39 28·92R	·2019750	+1318	−11 53·10
19	V	JP	70 76 15·10A	0 32·74T	·2116390	−2754	+ 0 36·54
Nov. 6	W, AC	AP, RC	97 63 41·78	3 38·09	·2036745	+1871	−11 48·56
7	JS, HF	C, AW	93 64 40·76	4 21·84	·2006463	+1294	−12 21·65
1906.							
Jan. 8	SD	AW	70 78 17·13T	6 35·68R	·2090273	−2032	− 2 57·08
9	AC	C	69 112 42·24A	37 9·98T	·2132070	−2167	+ 0 14·36
13	V	AP	78 106 31·98A	14 35·09T	·2218810	−1666	+12 16·07
Feb. 5	HF	AP, AW	69 119 12·94A	45 25·62	·2117330	−2535	− 1 14·93
Mar. 6	W, AC	C, M	70 98 26·41	18 7·38	·2185780	−3282	+ 3 51·73
7	JS	AP, AW	72 90 9·26T	4 48·09	·2252032	−3360	+ 7 18·61
Apr. 6	AC	C, AW	80 148 26·94A	52 38·20	·2394520	−3337	+14 7·30
May 7	E	C	98 137 1·94T	41 53·02T	·2534074	−1068	+14 59·69
June 7	JS	AW	109 124 41·00T	49 34·47T	·2493079	+2098	+ 2 21·86
July 2	E	AP, AW	103 102 9·45T	13 16·56	·2467262	−1464	+12 18·94
4	HF	AW	109 89 21·62A	11 19·95R	·2488160	+ 468	+ 4 53·27
Aug. 1	W	AP, J	109 124 25·31T	50 26·01	·2428892	+ 424	+ 2 18·21
2	HF	JP, M	109 123 4·24A	53 54·68	·2412740	+1187	− 2 11·79
30	JS	AW	109 81 20·47T	15 18·02R	·2346961	+1296	− 4 39·23
Sept. 27	W, AW	JP	108 68 52·70	5 22·40T	·2291990	+1812	− 7 2·47
28	E	AP	105 78 12·58T	16 33·83T	·2255307	+1922	−10 0·01
Oct. 8	AC	JW	69 80 3·57A	10 5·96T	·2001760	− 916	− 2 51·41
27	RC, AC	JP	98 116 4·69	55 47·59T	·2168470	+2253	−13 0·82

Change of $\delta_{\mathfrak{D}} - \delta_K$ Gr. – Cape.	Calculated Parallax. Greenwich.		Cape.		N.P.D. corrected for Parallax and Motion. Greenwich.			Cape.	57″·05 ÷ Sum of Parallaxes.	Correction to Hansen's Mean Parallax. Ellipticity = 1/293·5.	1/300.	Weight.
″	′	″	′	″	°	′	″	″		″	″	
−0·87	31	43·53	46	4·14	71	33	11·86	11·34	·734	+ ·38	+ ·01	1½
−1·13	35	42·64	45	35·62	74	51	55·61	55·02	·702	+ ·41	+ ·04	1½
−0·76	52	37·23	28	3·77	97	11	22·87	23·87	·707	− ·71	−1·10	1
−1·18	36	44·23	44	12·45	76	25	11·65	7·42	·705	+2·98	+2·61	0
−0·63	49	12·97	32	33·47	92	6	58·79	59·04	·698	− ·17	− ·54	1½
+0·51	52	6·03	17	43·62	105	35	60·20	56·89	·817	+2·70	+2·31	½
+0·39	53	15·39	16	46·16	106	51	22·64	20·03	·814	+2·12	+1·73	1
−0·39	55	14·95	16	38·53	108	42	47·04	48·32	·794	−1·02	−1·39	1
+0·04	54	44·93	15	2·09	108	56	10·83	9·00	·818	+1·50	+1·10	1
+1·27	46	18·62	26	17·49	95	25	48·16	48·31	·786	− ·12	− ·50	1
−0·06	54	30·04	14	53·79	109	1	17·23	14·94	·823	+1·88	+1·50	½
+0·08	53	50·02	15	3·81	108	39	28·00	27·00	·829	+ ·83	+ ·43	1
+1·17	47	52·23	23	44·65	98	25	10·38	8·85	·797	+1·22	+ ·84	1
+1·25	46	17·69	25	45·40	95	53	22·23	22·47	·792	− ·19	− ·57	½
−0·94	30	3·28	45	3·25	70	46	11·82	11·59	·760	+ ·17	− ·20	1
+1·16	47	10·43	24	40·05	97	16	31·35	30·74	·794	+ ·48	+ ·10	2
+1·28	44	48·24	27	49·29	93	19	52·52	50·76	·785	+1·38	+1·00	2
−0·51	29	54·15	44	44·01	70	48	22·98	22·10	·764	+ ·67	+ ·31	1
−0·87	29	50·34	45	26·87	70	22	51·90	50·34	·757	+1·18	+ ·81	1
−1·13	38	10·31	41	28·92	79	8	21·67	18·95	·716	+1·95	+1·57	1
−0·77	29	49·65	45	12·66	70	29	23·29	22·58	·760	+ ·54	+ ·17	1½
−1·28	30	50·82	45	36·12	71	7	35·59	33·95	·746	+1·22	+ ·86	2
−1·34	32	52·58	45	12·89	72	57	16·68	18·25	·731	−1·15	−1·51	1½
−0·88	41	10·98	40	31·47	81	47	15·96	16·09	·699	− ·09	− ·46	1½
+0·12	54	4·43	26	3·58	99	22	57·51	56·41	·712	+ ·78	+ ·41	1
+0·23	58	2·96	14	40·93	110	6	38·04	37·49	·784	+ ·43	+ ·05	1
+0·33	55	18·66	21	15·04	103	46	50·79	50·87	·745	− ·06	− ·45	1½
+0·43	57	47·46	15	18·99	109	31	34·16	32·64	·781	+1·19	+ ·80	1
+0·48	57	11·77	14	26·79	110	7	13·54	11·49	·796	+1·63	+1·24	1½
+0·47	56	58·60	14	20·19	110	6	5·64	3·55	·801	+1·67	+1·28	½
+0·51	55	53·94	14	45 81	109	25	26·53	25·11	·807	+1·15	+ ·74	1
+0·51	54	47·85	15	43·41	108	14	4·85	3·85	·810	+ ·81	+ ·41	1½
+0·61	53	18·68	18	18·13	105	24	53·90	52·56	·797	+1·07	+ ·68	1
−0·63	28	31·11	44	22·01	69	51	32·46	35·93	·783	−2·72	−3·08	0
+0·79	49	32·56	23	45·97	99	6	32·13	33·53	·779	−1·09	−1·48	1½

Date.	Observers. Gr.	Cape.	Apparent N.P.D. of Crater. Greenwich.	Cape.	Log sin p_K. Greenwich.	Corr. for Cape.	Motion of ☽ in N.P.D. Gr. – Cape.
1906			° ′ ″	′ ″			′ ″
Nov. 5	JS, AW	RC	68 101 24·75	29 52·89T	·1993872	– 926	– 1 3·58
1907.							
Jan. 26	E	RC	68 91 28·82T	20 15·06R	·1998190	– 814	– 1 23·51
Feb. 21	W	JW	69 121 19·50T	53 50·32R	·1994355	+ 200	– 5 27·62
22	JS, HF	M	68 102 54·96	32 48·92R	·1998266	– 506	– 2 31·21
Mar. 27	E, AW	AP	79 101 32·25	9 45·39R	·2208423	– 2791	+ 13 31·91
Apr. 23	JS, HF	AW	77 91 38·87	1 51·46T	·2165278	– 3143	+ 12 29·46
24	E, AW	JP	81 120 28·35	27 23·11T	·2229796	– 3315	+ 14 27·03
May 21	E, AW	JP	79 103 6·00	12 24·69T	·2159152	– 3253	+ 13 24·77
June 22	JS, AC	JP	101 150 34·89	59 42·40T	·2427442	– 3241	+ 14 18·18
Sept. 16	RC, AC	JP	111 120 26·93	51 23·36T	·2409552	– 1034	– 0 47·93
18	JS, BE	C	108 80 37·47	17 50·89R	·2427880	+ 166	– 9 52·02
27	JS, AC	C	68 119 37·43	52 2·20T	·2035986	+ 1680	– 5 48·90
Dec. 17	RC, AC	S	71 105 28·98	40 43·03T	·2065930	+ 1618	– 9 44·66
1908.							
Feb. 11	RC	AP	68 75 40·78T	8 14·52R	·2028721	+ 1768	– 5 38·75
14	E, BE	JW	67 101 18·16	24 52·79R	·1977740	+ 108	+ 4 20·11
15	RC, AC	RC	69 98 34·32	18 32·76R	·1980342	– 319	+ 7 19·89
Mar. 11	JS	AW	66 91 46·80T	19 38·95R	·2006191	+ 1225	– 0 4·54
14	RC, AC	JW	70 139 40·31	57 6·66R	·1994012	– 642	+ 9 13·17
23	JS, AC	M	110 99 27·75	22 17·03T	·2301514	– 2130	+ 7 54·67
Apr. 13	JS, AC	AP	81 98 40·04	8 30·29R	·2066033	– 1857	+ 14 26·85
14	E, AW	AW	86 96 37·07	5 4·34R	·2105957	– 2122	+ 15 30·14
May 13	RC, BE	AW	94 105 16·04	13 29·33R	·2188636	– 2802	+ 16 7·42
June 10	RC, AC	AW	96 149 46·02	59 0·67R	·2207657	– 3287	+ 15 45·11
11	W	C	102 91 49·74T	3 14·02T	·2273562	– 3323	+ 14 45·52
July 8	RC, HF	AP	99 134 25·42	45 35·82R	·2205147	– 3553	+ 15 1·91
Aug. 10	HF	AP	112 80 39·66A	15 50·51R	·2516339	– 1783	– 6 26·33
11	W	JP	109 80 53·16T	18 35·84T	·2538943	– 529	– 11 32·01
Sept. 5	E, BE	AW	113 99 42·58	30 1·22T	·2378973	– 3349	+ 1 32·65
7	HF	AW	111 70 28·47A	7 45·78R	·2491385	– 2010	– 8 58·03
12	E, BE	RC	83 121 12·09	57 38·81T	·2417210	+ 3290	– 18 5·15
14	W, AW	S	73 92 50·41	27 29·35T	·2272237	+ 3776	– 13 10·47
15	BE	JW	69 105 14·40A	38 39·72T	·2199241	+ 3582	– 9 49·96
Oct. 3	E	RC	113 112 14·16T	46 27·24R	·2349336	– 2686	– 1 41·40
6	BE	M	105 62 51·77A	2 40·96R	·2474836	– 1186	– 15 14·94
7	E, AW	AP	99 94 0·22	33 12·21R	·2488876	– 229	– 17 49·47

Change of $\delta_D - \delta_K$ Gr. – Cape.	Calculated Parallax. Greenwich.	Cape.	N.P.D. corrected for Parallax and Motion. Greenwich.	Cape.	57″·05 ÷ Sum of Parallaxes.	Correction to Hansen's Mean Parallax. Ellipticity = 1/293·5.	1/300.	Weight.
″	′ ″	′ ″	° ′ ″	″		″	″	
−0·87	27 56·72	44 39·09	69 13 28·03	27·53	·786	+ ·39	+ ·02	1½
−0·78	27 50·31	44 47·02	69 3 38·51	37·79	·785	+ ·57	+ ·20	1
−0·27	29 1·39	43 53·88	70 32 18·11	16·31	·782	+1·41	+1·04	1
−0·64	27 59·64	44 40·47	69 14 55·32	57·54	·785	−1·74	−2·11	1½
−0·91	38 9·33	40 3·81	80 3 22·92	20·20	·730	+1·99	+1·61	1½
−1·06	36 9·90	41 8·01	77 55 28·97	27·87	·738	+ ·81	+ ·44	1½
−0·74	40 2·06	38 35·38	82 20 26·29	24·78	·725	+1·09	+ ·73	1½
−0·95	37 44·64	39 34·58	80 5 21·36	23·09	·737	−1·28	−1·64	1½
+0·85	54 17·16	22 14·76	102 36 17·73	16·19	·745	+1·15	+ ·77	1½
+0·96	57 31·24	12 19·62	112 2 55·69	56·01	·817	− ·26	− ·65	1½
+0·47	56 36·46	16 0·41	108 23 61·01	59·75	·786	+ ·99	+ ·60	1½
−0·26	28 27·97	44 54·67	69 31 9·46	7·71	·778	+1·36	+ ·99	1½
−0·05	30 54·32	43 36·43	72 14 34·66	34·75	·766	− ·07	− ·44	1½
−0·30	27 49 11	45 14·09	68 47 51·67	49·56	·781	+1·65	+1·28	1
−0·97	27 1·59	45 4·32	68 14 16·57	16·25	·792	+ ·25	− ·12	1½
−1·05	28 37·57	44 4·63	70 9 56·75	56·23	·785	+ ·41	+ ·05	1½
−0·68	26 14·70	45 56·71	67 5 32·10	30·44	·791	+1·31	+ ·95	1
−1·00	30 3·54	43 17·32	71 49 36·77	36·15	·778	+ ·48	+ ·12	1½
+1·23	55 43·20	13 29·75	110 43 44·55	42·68	·825	+1·54	+1·16	1½
−0·58	38 18·08	37 23·86	82 0 21·96	20·42	·754	+1·16	+ ·79	1½
−0·31	41 59·00	34 3·93	86 54 38·07	38·10	·750	− ·02	− ·40	1½
+0·30	47 38·87	27 59·86	94 57 37·17	36·91	·754	+ ·20	− ·18	1½
+0·55	49 17·19	25 41·74	97 40 28·83	28·07	·761	+ ·58	+ ·20	1½
+0·96	52 24·30	21 24·96	102 39 25·44	25·46	·772	− ·02	− ·40	1
+0·76	50 34·52	23 11·00	100 23 50·90	49·49	·773	+1·09	+ ·70	1
+0·72	59 2·94	12 12·28	112 21 36·72	37·18	·801	− ·37	− ·76	1
+0·17	58 25·39	15 22·04	109 22 27·77	26·04	·773	+1·34	+ ·94	1
+1·42	57 33·41	10 33·78	113 42 9·17	9·07	·837	+ ·08	− ·32	1½
+0·53	58 22·14	13 18·83	111 12 6·33	7·11	·796	− ·62	−1·01	1
−0·45	43 16·70	38 22·58	84 17 55·39	55·79	·699	− ·28	− ·66	1½
−0·17	33 53·45	44 37·34	73 58 56·96	56·05	·727	+ ·66	+ ·29	1½
−0·13	30 11·97	46 12·17	70 15 2·43	1·80	·747	+ ·47	+ ·10	1
+1·17	57 13·03	10 12·91	113 54 61·13	59·92	·846	+1·02	+ ·63	½
−0·13	55 58·30	19 28·16	105 6 53·47	54·05	·756	− ·44	− ·83	1
−0·45	53 39·37	24 58·08	99 40 20·85	20·37	·725	+ ·35	− ·04	1½

Date.	Observers. Gr.	Cape.	Apparent N.P.D. of Crater. Greenwich.	Cape.	Log sin p_K. Greenwich.	Corr. for Cape.	Motion of ☽ in N.P.D. Gr. – Cape.
1908.			° ′ ″	′ ″			′ ″
Oct. 8	RC, AC	RC	93 86 24·48	24 32·62R	·2482677	+ 820	– 19 5·60
Nov. 3	E	RC	102 62 3·50T	2 28·24R	·2417000	– 683	– 16 29·56
6	W	M	83 118 8·01T	55 8·19R	·2400387	+1354	– 18 18·63
Dec. 10	RC, AC	C	65 114 38·97	37 3·30T	·2078298	+2118	+ 4 22·58
1909.							
Jan. 8	W	AW	69 100 14·36T	17 18·48T	·2020995	+1289	+ 9 30·02
30	HA, AC	M	69 92 40·34	26 39·91R	·2198777	+2528	– 10 19·62
Feb. 1	RC, AC	AW	65 76 46·58	5 47·79R	·2110298	+2009	– 2 32·60
Mar. 31	BE, AC	AW	69 86 14·86	3 21·57R	·2008910	+1398	+ 9 42·81
Apr. 30	RC, AW	JW	79 137 45·52	48 37·31R	·1985808	+105	+14 47·51
May 4	RC	RC	100 112 54·17T	26 52·73R	·2057572	– 1630	+15 3·28
29	W, AW	AP	87 146 44·91	56 46·22R	·2003725	– 715	+15 57·91
June 7	E, HF	S	112 76 13·16	17 4·36T	·2332021	– 1407	– 9 6·43
July 1	BE, AC	AP	113 77 5·60	3 39·40R	·2229259	– 2804	+ 7 13·28
Aug. 27	E	AP	114 122 42·05T	59 24·75R	·2331869	– 3699	– 2 54·54
Sept. 2	BE	S	85 75 13·52T	12 30·18T	·2484465	+1836	– 20 2·35
1910.							
Jan. 21	BE, AC	M	65 117 37·28	48 48·40R	·2332386	+1264	– 8 45·73
22	E	JJ	63 115 6·90T	42 30·41R	·2304225	+1612	– 3 43·85
30	RC, AC	JP	91 140 55·83	51 22·93T	·2002908	+ 682	+16 23·11
Feb. 21	W, HF	S	66 89 0·10	5 1·89T	·2167458	+1974	+ 8 55·62
22	E, AC	JP	69 121 13·99	33 34·76T	·2129363	+1918	+12 11·42
Mar. 1	BE, J	JJ	105 98 8·44	16 54·93R	·1998203	– 800	+13 37·28
2	HF	S	109 104 25·64A	27 28·60T	·2020995	– 1520	+11 27·68
29	BE	JJ	108 92 36·40T	14 25·87R	·1993135	– 841	+12 20·97
Apr. 1	AC	AW	116 71 31·62A	7 10·84R	·2097190	– 2709	+ 2 22·16
29	W, HF	RC	116 90 35·75	29 32·55R	·2096646	– 2401	– 0 30·84
June 16	RC, J	JP	93 112 50·39	23 28·63T	·2001476	+962	+16 37·11
18	BE, AC	C	103 130 23·44	47 37·32T	·1990625	– 304	+14 28·83
28	W	M	96 91 4·51T	33 12·18R	·2322015	– 2060	– 18 59·22
Aug. 19	RC, HA	RC	110 112 13·93	56 32·52R	·2269272	– 2652	– 12 31·76
Sept. 12	RC, AC	JP	116 92 14·08	28 4·85T	·2062245	– 2216	+ 2 59·75
20	RC	AP	83 68 28·11T	6 41·31R	·2465541	– 369	– 20 48·34
Nov. 14	E, HF	JW	83 87 7·55	25 22·54R	·2487951	– 2586	– 21 12·27
15	RC	C	76 99 3·52T	35 16·37R	·2528407	– 1497	– 19 51·85
Dec. 15	AC	RC	64 79 51·35A	8 27·59T	·2528805	+381	– 9 9·43

Change of $\delta\mathfrak{D}-\delta_K$ Gr.–Cape.	Calculated Parallax. Greenwich.		Cape.		N.P.D. corrected for Parallax and Motion. Greenwich.			Cape.	57″·05 ÷ Sum of Parallaxes.	Correction to Hansen's Mean Parallax. Ellipticity = 1/293·5.		1/300.	Weight.
″	′	″	′	″	°	′	″	″					
−0·61	50	12·84	30	44·08	93	36	11·64	10·49	·705	+	·81	+ ·43	1½
−0·23	53	56·54	22	9·64	102	8	6·96	8·09	·750	−	·85	−1·24	1
−0·59	43	4·46	38	14·65	84	15	3·55	3·62	·702	−	·05	− ·43	1
−0·61	26	9·52	47	5·43	66	28	29·45	30·70	·779	−	·97	−1·34	1½
−0·67	28	55·08	44	31·19	70	11	19·28	19·02	·777	+	·20	− ·16	1
−0·30	30	1·19	46	18·17	70	2	39·15	38·16	·747	+	·74	+ ·37	1½
−0·42	25	48·54	47	42·22	65	50	58·04	56·99	·777	+	·82	+ ·45	1½
−0·50	28	38·95	44	31·52	69	57	35·91	35·40	·779	+	·40	+ ·04	1½
−0·35	36	40·28	37	38·86	80	41	5·24	3·33	·767	+1·46		+1·10	1½
+0·56	49	10·02	21	48·73	101	3	44·15	45·30	·804	−	·92	−1·31	1
−0·05	42	7·89	31	51·93	88	44	37·02	36·01	·771	+	·78	+ ·39	1½
+0·58	56	34·50	11	40·69	112	19	38·66	39·20	·836	−	·45	− ·84	1½
+1·39	55	30·82	10	38·31	113	21	34·78	32·38	·862	+2·07		+1·67	1½
+1·10	57	15·80	8	56·56	115	5	26·25	27·87	·862	−1·40		−1·79	1
−1·07	44	50·88	37	56·07	85	30	22·64	22·83	·689	−	·13	− ·50	1
−0·65	27	46·77	49	48·70	66	29	50·51	50·72	·735	−	·15	− ·52	1½
−0·43	25	45 14	50	35·94	64	29	21·76	22·07	·747	−	·23	− ·59	1
+0·13	44	22·22	28	46·99	92	36	33·61	33·16	·780	+	·35	− ·02	1½
+0·02	27	11·86	47	49·26	67	1	48·24	46·79	·761	+1·10		+ ·75	1½
+0·05	29	56·29	45	29·79	70	31	17·70	16·02	·756	+1·27		+ ·90	1½
+0·62	50	21·95	17	14·03	105	47	46·49	46·86	·846	−	·31	·70	1½
+0·85	51	58·26	13	29·98	109	52	27·38	27·11	·872	+	·24	− ·16	1
+0·76	51	16·65	14	31·85	108	41	19·75	19·45	·868	+	·26	− ·13	1
+1·21	54	29·06	7	23·33	116	16	62·56	57·54	·922	+4·63		+4·24	0
+1·10	54	32·29	7	1·78	116	36	3·46	4·59	·927	−1·05		−1·45	1
+0·25	45	10·88	27	31·60	94	7	39·51	37·59	·792	+1·52		+1·14	1½
+0·54	49	44·47	18	32·28	104	20	38·97	38·97	·835	...		− ·38	1½
−0·65	50	5·48	26	46·84	96	40	59·03	59·15	·742	−	·09	− ·47	1
0·00	55	22·22	12	49·94	110	56	51·71	50·70	·837	+	·85	+ ·44	1½
+0·93	54	6·77	6	59·86	116	38	7·31	5·39	·934	+1·79		+1·36	1½
−1·50	43	6·98	39	27·52	83	25	21·13	18·99	·691	+1·48		+1·11	1
−1·68	43	34·22	39	23·45	83	43	33·33	32·04	·688	+	·89	+ ·51	1½
−1·77	38	35·59	45	3·73	77	0	27·93	26·48	·682	+	·99	+ ·62	1
−0·98	27	30·87	53	2·18	64	52	20·48	19·36	·708	+	·79	+ ·43	1

Notes.

1905 April 14. The Greenwich observation appears to be 3″ in error by comparison with the observation of the limb and observations of neighbouring days.

1905 May 23. The mean N.P.D. of the comparison stars at Greenwich is 18° North of the Moon.

1905 Aug. 11. Only one comparison star observed at Greenwich, 35° North of the Moon.

1905 Oct. 10. No comparison stars were observed at Greenwich on this day. The equator-point was deduced from star observations on neighbouring days.

1906 Aug. 2. Same note as 1905 Oct. 10.

1906 Oct. 8. Rejected on account of the large outstanding residual. The Greenwich observation does not appear to be much in error by comparison with neighbouring observations.

1908 July 8. This day is only given unit weight because it has been necessary to include stars differing considerably from the Moon in N.P.D.

1908 Oct. 3. This day is only given half weight because the G.M.T. of the Cape observation was only given to the nearest minute. It has been assumed in the reduction that the time should correspond to the meridian passage of the crater. The rate of motion in N.P.D. is very small.

1910 Apr. 1. Same note as 1906 Oct. 8.

1910 Apr. 29. Same note as 1908 July 8.

Giving the weights indicated in the table, the resulting corrections to Hansen's Mean Parallax are

$$(1) \quad +0''{\cdot}50 \qquad\qquad (2) \quad +0''{\cdot}12$$

A mean was also taken giving each day equal weight, except those for which the weight 0 is entered; the result was

$$(1) \quad +0''{\cdot}52 \qquad\qquad (2) \quad +0''{\cdot}14$$

Hence it appears that the result will be appreciably the same for any reasonable system of weighting the separate days.

For simplicity the equal-weight solution was adopted in finding the probable error of a single determination, and for examining the distribution of the errors. The following table gives the corrections to Hansen's Parallax on Hypothesis (1) diminished by their mean value 0″·51, and arranged in descending order of magnitude :—

″	″	″	″	″
+2·19	+0·72	+0·30	−0·16	−0·70
1·60	0·72	0·28	0·25	0·74
1·56	0·69	0·28	0·25	0·77
1·48	0·67	0·27	0·27	0·77
1·42	0·65	0·23	0·31	0·83
1·39	0·64	0·17	0·31	0·88
1·16	0·63	0·15	0·33	0·95
1·14	0·59	0·07	0·43	0·96
1·12	0·58	0·05	0·51	1·13
1·04	0·57	+0·03	0·53	1·21
1·03	0·55	−0·02	0·53	1·36
0·99	0·51	0·02	0·56	1·43
0·99	0·49	0·04	0·57	1·48
0·96	0·48	0·08	0·58	1·53
0·90	0·38	0·10	0·60	1·57
0·89	0·34	0·10	0·60	1·62
0·86	0·32	0·11	0·63	1·65
0·82	0·31	0·12	0·64	1·79
0·81	0·31	0·13	0·67	1·90
−0·76	+0·30	0·14	−0·68	−2·26
Sum of columns without regard to sign	23·11	10·45	2·69	9·41	25·53

Total Sum . . . . 71″·19.
Mean error of one observation, 0″·712.
Mean error of 100 observations, 0″·071.
Probable . . . . 0″·06.

It will be noticed that the number of + and − residuals is exactly equal, and that the distribution of the errors is fairly symmetrical. Hence the theoretical value of the probable error appears to be applicable.

The only source of systematic error to which the method appears to be liable is that arising from personality in bisecting the crater. To investigate the amount of personality the resulting parallax corrections on hypothesis (1) were arranged for the different observers, selecting five different Greenwich observers and six different Cape observers. The results are—

Greenwich Observers.			*Cape Observers.*		
	″	No. of Observations		″	No. of Observations.
AC	+0·33	28	RC	+0·22	14
RC	+0·52	20	C	+0·86	12
HF	+0·28	16	M	+0·15	11
JS	+1·00	13	AP	+0·77	17
W	+0·46	16	JP	+0·51	15
			AW	+0·37	22
Weighted mean	+0·48			+0·48	
Simple mean	+0·52			+0·48	

Remembering that the probable error of the mean of sixteen observations is 0″·16, the discordances from the mean are not larger than we should expect. In any case it appears unlikely that the effect of individual personalities on the general mean is appreciable, for even if JS, the observer with the largest apparent personality, were omitted, the resulting general mean would only be diminished by 0″·04, or considerably less than the probable error. If, however, there is a general tendency both at Greenwich and the Cape to bisect the crater either too high or too low, this will have an effect on the parallax, since the Moon is always south at Greenwich and north at the Cape. The above observations do not afford the means of testing for this possible source of error. It will only be eliminated by the use of a reversion prism eyepiece; the use of such an eyepiece would, however, render the identification of the crater difficult. It is by no means too easy to rapidly identify it under existing conditions, and it would be considerably harder if the familiar configurations were reversed.

Determination of Figure of Earth.

It will be seen that the greater part of the uncertainty in the observational determination of the Moon's equatorial Horizontal Parallax is that which arises from uncertainty in the ellipticity of the Earth. This source of uncertainty also enters, though to a less extent, into the gravitational determination of the Parallax. This involves a knowledge of gravity at the latitude whose sine is $\frac{1}{\sqrt{3}}$, when the Earth attracts as though its mass were concentrated at the centre; and the distance of the Moon is determined primarily in terms of the Earth's radius at this latitude. Consequently the parallax involves the ratio of this radius to the equatorial radius.

The formula for the determination of the Moon's parallax by the gravitational method is—

$$\frac{a_1^3}{a^3}\sin^3\pi = \frac{1}{1+\mu}\,\frac{n^2 a_1}{g_1}\left(1+\frac{n'^2}{2n^2}\right),$$

where π is the Moon's mean parallax.

μ the ratio of the mass of the moon to the mass of the Earth.

n the Moon's mean motion.

n' the Earth's mean orbital motion.

a_1 and g_1 the radius of the Earth and gravitational attraction in latitude $\sin^{-1}\frac{1}{\sqrt{3}}$.

a_1 the Earth's equatorial radius.

The factor $1+\frac{n'^2}{2n^2}$ represents a small correction due to the disturbing effect of the Sun on the Moon's motion.

Also, if e is the ellipticity, we have (neglecting e^2)

$$\frac{a^3}{a_1^3} = 1 + e.$$

To examine the effect on the parallax of small changes in the adopted data, we differentiate and insert numerical values in the coefficients—

$$d\pi'' = +0\cdot18 da_1 - \cdot013 d\left(\frac{1}{e}\right) - 1\cdot2 dg_1 + 0\cdot17 d\left(\frac{1}{\mu}\right).$$

where a is measured in kilometres and g in C.G.S. units.

Newcomb (*Astronomical Constants*, p. 194) gives a correction to Hansen's mean parallax $+0''\cdot45$, based on the data—

Newcomb adopted.	*Most recent values.*
$a_1 = 6371\cdot004$	$6370\cdot843$*
$g_1 = 979\cdot77$	$979\cdot76$†
$\frac{1}{\mu} = 81\cdot45$	$81\cdot53$‡
$\frac{1}{e} = 293\cdot5$	

The substitution of the more recent values of a_1, g_1, and $\frac{1}{\mu}$ makes no change in π, as the small corrections cancel one another. Thus by the gravitational theory the correction to Hansen's mean parallax is

$$\delta\pi = +0''\cdot45 - 0''\cdot013 d\left(\frac{1}{e}\right).$$

The results of the observations of Mösting A may be written

$$\delta\pi = +0''\cdot49 - 0''\cdot057 d\left(\frac{1}{e}\right).$$

$\frac{1}{e} = 293\cdot5$ being taken as standard.

Solving the two equations,

$$\frac{1}{e} = 294\cdot4.$$

$$\delta\pi = +0''\cdot44.$$

The following table gives the correction to Hansen's Mean

* Newcomb, *Spherical Astronomy*, Appendix, Helmert's result.
† Helmert, from pendulum observations (1901).
‡ Hinks, *M.N.*, lxx.

Parallax for different values of the ellipticity, as deduced (1) from observation, (2) from gravitational theory :—

$\frac{1}{e}$	From Observation.	From Gravitation.
293	+ 0".53	+ 0".46
294	0·47	0·45
295	0·41	0·43
296	0·35	0·42
297	0·29	0·40
298	0·24	0·39
299	0·18	0·37
300	+ 0·12	+ 0·36

The value of $\frac{1}{e}$ which will make the two columns agree is $294{\cdot}4 \pm 1{\cdot}5$. This is a determination of ellipticity entitled to tolerable weight. When South Africa is geodetically connected with Europe there will be a new independent value of $\frac{1}{e}$ applicable to the mean meridian of Africa, and the observational parallax will be cleared of this source of uncertainty (at least as regards the equatorial parallax in that meridian ; but if the equator is not an accurate circle, the equatorial parallax will vary with the meridian).

In the Greenwich Lunar Reductions, Hansen's Parallax is employed unaltered, the value of $\frac{1}{e}$ being taken as 300. The above table shows that the use of these constants only leads to an error of $0''{\cdot}12 \sin ZD$ in the deduced geocentric place of the Moon.

The discussion of the observations has been made by Mr. Crommelin.

Discussion of the Greenwich Reflex Zenith-Tube Observations 1906–9. By A. S. Eddington, M.A., M.Sc.

(*Communicated by the Astronomer Royal.*)

It is well known that the series of observations with the Greenwich Reflex Zenith-Tube, starting in 1856 and continued to 1899, led to results which at the time seemed discordant and inexplicable. Mr. Chandler, however, showed (*Astron. Jour.*, No. 511) that the anomalous results were such as might reasonably be attributed to latitude-variation, and that the observations might be used to determine the amount of this variation. Accordingly work with the instrument was resumed in 1902 with a much extended programme, and has been continued up to the present time.

The object of the present investigation is to find from the observations now accumulated how far the instrument is capable of giving reliable and useful results, and whether it is desirable to continue the work. I have therefore entered into a criticism of the instrument, showing some disadvantages inherent in it, which have appeared in the course of the work.

The observations from 1907–9 were first completely discussed; the results for 1906 were added later, and are based on revised star-places deduced from the final results of the 1907–9 observations. Material is also available for the years 1903–5, but the observations were not so numerous or so well distributed as in the later years.

The number of observations in each year is as follows:—

1903	781	1907	1638
1904	817	1908	1107
1905	799	1909	1302
1906	1170	1910	1362

The stars observed are about 80 in number, passing within 55′ of the zenith of Greenwich, and ranging down to about the seventh magnitude. The instrument measures the zenith-distances of these stars; and, by comparing the observed zenith-distances with the adopted star-places, residuals are found which should give the latitude-variation. The most difficult part of the discussion is the determination of the mean star-places, which are obtained by a series of successive approximations from the observations themselves. Much difficulty is caused by a scale-correction to the zenith-distances, which varies from night to night, and must be determined each night from the observations. Further, the limited number of stars within range of the instru-

ment renders it impossible to devise a symmetrical programme, and adds to the labour and intricacy of the discussion. These difficulties will be more fully explained later; we shall first state the results of the investigation.

I have not attempted to determine any correction to the constant of aberration (20″·47): as will presently appear, the observations are hardly sufficiently trustworthy to make the attempt worth while.

The results for latitude-variation are given in the next table, and are compared with the provisional results from the International latitude stations published annually by Professor Albrecht. The means given are for groups of nights, the extent of each group being adjusted so as to give determinations of equal weight.

Latitude-Variation at Greenwich. (*Groups of equal weight.*)

Date.	Latitude-Variation. Reflex Zenith-Tube.	International Result.	Diff.
1906.	″	″	″
Jan. 25	− ·26	− ·06	− ·20
Mar. 20	− ·18	+ ·10	− ·28
June 1	− ·11	+ ·18	− ·29
July 10	+ ·11	+ ·15	− ·04
Aug. 20	− ·17	+ ·08	− ·25
Sept. 5	+ ·04	+ ·05	− ·01
Oct. 1	+ ·06	+ ·01	+ ·05
Nov. 10	− ·07	− ·04	− ·03
Dec. 15	− ·08	− ·07	− ·01
1907.			
Jan. 30	+ ·06	− ·05	+ ·11
Mar. 1	+ ·09	− ·03	+ ·12
Mar. 25	+ ·13	− ·02	+ ·15
May 1	− ·15	+ ·01	− ·16
June 25	+ ·03	+ ·04	− ·01
July 25	+ ·07	+ ·06	+ ·01
Aug. 25	+ ·07	+ ·07	·00
Sept. 10	+ ·24	+ ·09	+ ·15
Sept. 20	+ ·14	+ ·09	+ ·05
Oct. 1	+ ·13	+ ·10	+ ·03
Oct. 20	+ ·05	+ ·11	− ·06
Dec. 1	+ ·24	+ ·10	+ ·14

Latitude-Variation at Greenwich—continued.

Date.	Latitude-Variation. Reflex Zenith-Tube.	International Result.	Diff.
1908.	"	"	"
Jan. 15	+ ·09	+ ·04	+ ·05
Feb. 15	+ ·04	− ·05	+ ·09
Apr. 10	− ·05	− ·17	+ ·12
June 5	− ·17	− ·20	+ ·03
July 20	− ·16	− ·13	− ·03
Aug. 20	− ·15	− ·04	− ·11
Sept. 5	− ·04	·00	− ·04
Oct. 5	+ ·03	+ ·08	− ·05
Nov. 20	+ ·21	+ ·18	+ ·03
1909.			
Jan. 30	+ ·57	+ ·20	+ ·37
Apr. 10	+ ·09	+ ·02	+ ·07
May 10	− ·16	− ·10	− ·06
June 10	− ·11	− ·19	+ ·08
July 20	− ·21	− ·29	+ ·08
Aug. 10	− ·38	− ·32	− ·06
Sept. 1	− ·36	− ·32	− ·04
Sept. 20	− ·11	− ·28	+ ·17
Oct. 15	− ·23	− ·20	− ·03
Nov. 20	+ ·15	− ·02	+ ·17

It will be noticed that there are often large discordances between the Reflex Zenith-Tube results and those of the International determination. (The latter may be taken to be practically correct.) At first sight it seems as if the latitude-variation is entirely masked by unknown errors. This is the case in 1906, but there is an improvement in the later years; and, whilst the results cannot be considered satisfactory, the rather large oscillations in 1908 and 1909 are fairly correctly indicated.

From the differences in the last column of the table the probable error of the results given is $\pm 0''{\cdot}079$. If we ignored latitude-variation and treated the results as simple errors of observation, the probable error would be $\pm 0''{\cdot}123$. The reduction is very considerable when it is remembered that during half the time, 1906 and 1907, the range of latitude-variation was very small. Taking 1908 and 1909 alone, the probable error is reduced from $\pm''{\cdot}147$ to $\pm''{\cdot}074$.

The discordances are, however, much greater than could arise from the accidental errors of observation. Calculation shows that the quantities in the above table should be subject to an accidental

error of $\pm 0''\cdot04$,* about half the error actually found. This confirms an opinion, formed whilst making the reductions, that considerable systematic errors may affect individual nights or longer periods. A conspicuous instance of this is afforded by the observations of 1909 January and February, which show a discordance of $0''\cdot37$. This discordance seems to be quite systematic throughout the period. The difficulty of obtaining precise information as to these errors is mainly due to the complication introduced by the scale-correction, which will now be considered.

Scale-correction.—There is no means of ensuring constancy of scale in measuring the zenith-distances with the Reflex Zenith-Tube, and the actual value of the micrometer-screws on any particular night can only be found from the observations of stars. The light of a star, after passing downwards through the object-glass, falls on a trough of mercury, is reflected, and comes to a focus just above the object-glass in the plane where the micrometers and wire frame are placed. Any change in the distance between the object-glass and mercury surface will accordingly change the value in arc of the micrometer and wire intervals, because it changes the length of the path of the ray from the object-glass to the plane of the micrometer. It is not possible to make any sort of rigid connection between the mercury trough and object-glass; in practice the trough rests on a pad of rubber sponge in order to avoid vibration as far as possible. A focussing rod is provided which enables the distance to be maintained roughly constant; as, however, a change of $\frac{1}{100}$ inch would involve a correction of $0''\cdot10$ to the zenith-distance of a star at $10'$ from the zenith, and over half a second to that of an extreme star such as β Draconis, it is clear that there will be large residual corrections for scale.†

The following table gives the correction for scale deduced from the discussion of the observations during the years 1907–8–9. For brevity, only the mean correction during a long group of nights is given, as this will show sufficiently well the general magnitude of the corrections that must be applied to the results printed in the Greenwich volumes: in the actual reductions, however, the scale correction was determined and applied for each night individually, because, notwithstanding a certain amount of continuity, there are often sudden changes and exceptional nights.

* This is found as follows. Each group corresponds to 130 observations on the average. As the fainter stars are given half-weight, and allowance must be made for loss of weight due to the asymmetry of the stars observed (see below—*scale-correction*), this is equivalent to about 80 full-weight observations symmetrically distributed. A table is given later of the probable errors of individual observations for a number of stars; the average for full-weight stars is about $\pm''\cdot36$. Thus, if there were no systematic error, the mean of the group would have a probable error $\pm''\cdot04$.

† The instrument was originally designed for observing γ Draconis, which transits so near to the zenith that the scale-correction could hardly be appreciable.

Mean Values of Corrections for Change of Scale.

Correction per 10′ Z.D.

Date.	Corrn.	Date.	Corrn.
1907. Jan. 1–Feb. 11	+″·027	1908. June 24–Aug. 10	+″·145
Feb. 15–Mar. 11	+·093	Aug. 11–Aug. 28	+·113
Mar. 14–Mar. 27	+·038	Aug. 29–Sept. 19	+·119
Apr. 1–May 28	−·020	Sept. 25–Oct. 15	+·154
June 1–July 16	−·028	Oct. 21–Dec. 31	+·218
July 17–Aug. 10	−·318	1909. Jan. 6–Feb. 22	+·270
Aug. 19–Sept. 2	−·235	Mar. 17–May 3	+·381
Sept. 3–Sept. 13	−·185	May 4–May 20	+·427
Sept. 16–Sept. 23	−·131	May 21–July 1	+·400
Sept. 24–Oct. 4	−·077	July 8–July 30	+·404
Oct. 7–Oct. 30	−·140	July 31–Aug. 27	+·421
Oct. 31–Dec. 31	−·057	Aug. 28–Sept. 9	+·385
1908. Jan. 1–Feb. 12	−·074	Sept. 10–Sept. 30	+·433
Feb. 14–Mar. 11	+·080	Oct. 6–Nov. 5	+·380
Mar. 18–May 10	+·128	Nov. 6–Dec. 31	+·357
May 13–June 23	+·075		

To take an extreme example, the printed zenith-distances of β Draconis (53′ N.) require average corrections of—

−1″·65 during 1907 July 17–Aug. 10.

+2″·32 during 1909 Sept. 10–Sept. 30.

The corrections for scale were determined each night by comparing the residuals of the more extreme north and south stars. If the distribution of the stars observed on any night is fairly symmetrical about the zenith, only a small and well-determined correction has to be applied to the mean result of the night, and the latitude-variation is determined without much additional error from this cause. Unfortunately, the distribution of the stars near the zenith of Greenwich is extremely unsymmetrical and unfavourable. For example, from R.A. $17^h\ 28^m$ to $19^h\ 31^m$ (which, owing to the presence of the bright stars β and γ Draconis and ι^2 Cygni, observable in daylight, is a very important group) there are available 14 stars north of the zenith and 5 south; the mean position of these 19 stars is Z.D. 16′ N., so that a large correction must be applied. Curiously enough, there is a similar scarcity of south stars in the other groups. Of stars of Z.D. greater than 30′ (suitable for determining scale corrections) the whole working list includes 23 north and only 8 south. This is due to the actual distribution of the stars in the sky; it does not appear possible to find any suitable south stars to redress the balance. It is only during the months October to January that it has been found possible to obtain symmetry in the observations.

When the resulting latitude-variation for a night is discordant, it is often difficult to discover whether the error is due to a systematic error of all the stars or is due to an error in the scale. This is especially the case when, as frequently happens, the determined scale-correction depends (at one end) on a single south star.

Determination of the mean star-places.—Naturally, it is not practicable to spread the observations with the Reflex Zenith-Tube over the whole of the stars passing the zenith of Greenwich. The scheme of observing must be arranged so that there may be a fairly large number of observable stars, avoiding the long intervals with few or no suitable stars. Also it is desirable that the few very bright stars, which can be observed on nights of poor definition, in twilight or in daylight, should be included. The stars observed at Greenwich fall into two main groups—(A) $4^h\ 59^m$–$9^h\ 26^m$, and (B) $17^h\ 28^m$–$22^h\ 34^m$, with two subsidiary groups, (*a*) $12^h\ 19^m$–$14^h\ 22^m$ and (*b*) $0^h\ 12^m$–$1^h\ 44^m$—the last mentioned consisting of only three stars.

The general principle in determining the star-places is, first, to compare the individual stars of a group with one another, and obtain a series of star-places consistent throughout the group; secondly, to compare results from two groups when they are observed at the same time of the year, and obtain corrections to reduce the star-places of one group to the system of the other. It is here that the necessity for the scale-correction presents great difficulties. Although the process is somewhat disguised in the actual reductions, we have virtually to connect together the places of the north stars and of the south stars independently: as already mentioned, the south stars are few in number, and this part of the connection is accordingly weak.

There is also the possibility that the scale may change (perhaps systematically) during an evening. This would introduce errors of a serious character.

It is not possible to proceed in any really systematic way: we must gradually build up a system of star-places, altering our methods according to the data available. Generally two or three successive approximations were necessary. The places of the stars of group B were the most satisfactorily determined. For group A the observations were much less numerous and less conveniently distributed. I found it impossible to adhere to my original plan of making the places depend on the Reflex Zenith-Tube observations alone, and had to make use of the known latitude-variation (from the international determination), and in some cases Professor Boss's catalogue. Rather less than half of the stars are included in the latter catalogue; but this was only used as a last resource, as there are theoretical objections to the use of places not determined with the instrument itself.

After calculating the latitude-variation and scale-corrections for 1907–9, a revision of the star-places was made, the corrected residuals having been tabulated. It was soon found that the places

of group B were practically unaltered, and the revision of these was not proceeded with. The remaining groups were completely revised; the corrections amounted generally to about 0″·10, ranging occasionally up to one-third of a second or more. Logically the latitude-variation from 1907–9 should now be redetermined with the new star-places, as some alteration would probably be made. The process, however, of alternate revision of the latitude-variation and of the star-places would be interminable; and, as it did not appear that the discordances would be removed by using the new places, I contented myself with using them to reduce the observations of 1906 as a test of their value. The result rather suggests that the original star-places may be generally better than the revised.

From this revision of the star-places I extract the following table, which will give an idea of the number of observations available, agreement of the separate years 1907–8–9, and probable errors of observations:—

Star.	Mag.	Zenith-Distance N.	Corrn. to Adopted Dec. from 1907.	1908.	1909.	Mean.	No. of Obs.	Mean Error of 1 Obs.
		′	″	″	″	″		″
γ Draconis .	2·4	+ 1	+ ·02	+ ·06	− ·04	+ ·01	209	± ·32
Gr. 2559 .	6·8	+ 1	− ·07	+ ·14	− ·10	− ·03	55	·51
Pi. XIX. 211	5·6	− 27	+ ·14	+ ·20	+ ·02	+ ·10	69	·38
B.D. 50° 3401	6·6	− 26	− ·15	+ ·15	+ ·19	+ ·03	37	·47
π^1 Cygni .	4·8	− 43	− ·03	− ·13	+ ·13	− ·01	67	·39
β Lacertæ .	4·6	+ 17	− ·08	+ ·03	+ ·11	+ ·01	82	·33
9 Lacertæ .	4·8	− 25	+ ·14	+ ·26	− ·37	+ ·02	82	·41
9 Aurigæ .	4·9	0	+ ·07	+ ·09	+ ·23	+ ·12	58	·38
27 Lyncis .	4·9	+ 18	+ ·10	+ ·21	+ ·13	+ ·14	56	·40
θ Boötis .	3·9	+ 49	+ ·14	− ·02	+ ·09	+ ·07	73	·47

The point of special interest is the discordance of the positions of 9 Lacertæ from the 1908 and 1909 observations. I have not come across any other discordance so definite as this, although there is often a suspicion of a sudden shift occurring in other star-places. The star B.D. + 50° 3401, at nearly the same zenith-distance, and transiting about an hour before 9 Lacertæ, serves as a check, and shows that the scale-correction and the wire-intervals are not at fault. The only cause I am able to suggest is that possibly there is an irregularity in the object-glass through which the rays forming the image of 9 Lacertæ passed during 1909. The second passage of the light through the object-glass, when the pencil of rays is very narrow and near a focus, is evidently an unsatisfactory feature of the instrument. In consequence of precession, the precise part of the object-glass through which the ray passes continually changes; and it seems possible that irregularities in the density of the glass might be sufficient to distort the path of

the ray from time to time.* With a view to testing this hypothesis, I examined the residuals of 9 Lacertæ separating the observations in the two positions of the object-glass. The results were—

	1907.	1908.	1909.
Position R	+″·15	+″·38	−″·54
Position L	+·08	+·02	−·20

This is somewhat indecisive; but, having regard to the probable error, the results from position L are fairly accordant; and the whole discordance may perhaps be attributed to an irregularity in the part of the object-glass through which the rays pass in position R. It would be interesting to see whether another star suffers a similar distortion in passing this particular position; but there is no other star observed in precisely the right place.

Among the minor difficulties may be mentioned the following:—

Determination of the wire-intervals.—The wire system consists of 33 parallel wires extending over a space of about $3\frac{1}{2}$ inches (105′). The measurement of the intervals is made by means of the micrometer-screws. Although the determination of the individual intervals appears to admit of very high accuracy, judging by the accordance of the measures, the distances between the extreme wires must be liable to cumulative error. Thus a discontinuity may occur whenever the wire system is renewed.

Inclination of the wires.—It is hardly probable that in such an extended system of wires an exact parallelism of all the wires should be obtained. An inclination of the wires would be shown by a systematic difference in the measured position of a star according as it was observed with the object-glass first in position L, then R, or *vice versa*. The complication introduced by the scale-corrections renders it difficult to detect such an effect directly.

Illumination of the field of view.—Prior to 1909 the field was illuminated by an annular reflector, and the brightness varied by tilting the reflector. It was found, moreover, by Sir William Christie, that the annulus cut off part of the pencil of rays for stars of all but the smallest zenith-distances, so that the full aperture of the object-glass was not used. Thus in the years 1906–7–8 the observations suffer from the unsatisfactory nature of the illumination. At the beginning of 1909 a rheostat was introduced to control the illumination, and the annulus was replaced by two long parallel strips of opal glass running horizontally north and south, and allowing the full pencil of light from a star of any zenith-distance (on the meridian) to pass between them. This arrangement is found to work well in practice, and seems to be free from all theoretical objection.

Site of the instrument.—The Reflex Zenith-Tube is placed in a

* The object-glass is nearly a hundred years old; it originally belonged to Troughton's transit instrument.

rather unfavourable position. Had a better site been available for it, the results might have been more satisfactory.

It is difficult to draw a general conclusion from this experience of the working of the instrument; there is at least a partial success, and it is possible that further improvement may be made; but it seems very doubtful whether the present instrument is capable of giving results up to the standard of accuracy which has been set by recent latitude-variation research. The fact that we have practically no choice as to what stars shall be observed, and that it is therefore not possible to arrange the observations to eliminate the uncertain corrections, heavily handicaps the Reflex Zenith-Tube as compared with other latitude instruments. It must be remembered, too, that the standard to be attained has been raised greatly since this series of observations was started. The extraordinary accuracy now reached with the Zenith-Telescopes of the International Latitude-Service puts into the background results which a few years ago might have been of much interest.

With an instrument of larger aperture capable of being used with faint stars, the particular difficulties which we have found should mostly disappear, especially if the instrument is designed so as to avoid the second passage of the light through the object-glass. With sufficient stars available, the observations can be made more symmetrical; but the scale correction must always present a difficulty when we come to determine the mean places of the stars from the observations. It may be best to determine the variations of scale several times each evening by rotating the object-glass and wire frame through 90°, and observing transits of central stars over a few of the outermost wires on each side: it would be necessary at the same time to rotate the arrangement for illuminating the field of view through 90° in order to avoid a partial obstruction of the pencil of rays. But this method could only be satisfactorily applied to a larger instrument than the one at Greenwich, as the number of stars whose transits can be observed is not large enough.

Royal Observatory, Greenwich:
1911 *May* 12.

Discussion of Occultations observed during the Lunar Eclipse of 1898 *December* 27. By Walter Heath, M.A.

Dr Backlund in the year 1900 published a list of these observations, 474 in number, with a catalogue of the stars based on two photographs taken at Pulkovo: the table below contains the results of a reduction of 363 of the observations.

The number of stars occulted was 65; 47 of them are included in plate 569 (Paris) of the *Catalogue Photographique*, and 30 (including those occulted at the Cape and Durban) are in plate 870.

I have endeavoured to use these plates as a check on the Pulkovo Catalogue, and the corrections adopted as the result of the comparison are given in the catalogue below.

I wish to thank M. Lagarde for having assisted me with his advice on the method of comparison. The inquiry was restricted to the details of the group of occulted stars on each plate, and the plate constants were found from the Pulkovo positions of these stars. The right ascension found for the centre of plate 870 agreed with the value given in the *Éléments Provisoires*, but in the case of plate 569 the value found exceeded that given in the *Éléments* by $0^s{\cdot}032$. I have applied no correction to the Pulkovo places in respect of this discordance.

The numbers and positions given for the stars numbered in the list below are taken from the Pulkovo Catalogue, but the positions of the additional stars denoted by the letters a to k are taken from the *Catalogue Photographique*, using the plate constants above referred to.

Hansen's value of the parallax was at first used in the reduction; but, at the suggestion of Dr. Crommelin, the results have been corrected by adopting Newcomb's value, $57'\ 2''{\cdot}68$, and a corresponding slight change in the Moon's geocentric declination was also allowed for. The corrections assumed for the lunar tables of the *Nautical Almanac* were

$$\Delta\alpha = +0^s{\cdot}394 \qquad \Delta\delta = +1''{\cdot}84.$$

Dr. Helmert's value of the compression, $1/298{\cdot}2$, has been used, and a correction for refraction was applied in the case of the observations made in America and in the southern hemisphere.

Since the date of Dr. Backlund's pamphlet certain corrections of longitude have been adopted at Toulouse, Heidelberg, and the Flower Observatory, giving improved values in the present table.

The probable error in some cases was increased by the bad weather prevailing in Western Europe; and, as a rule, I have omitted large positive residuals: they may possibly represent either (1) an interval during which the star happens to be obscured by cloud, or (2) an interval during which the star after emersion happens to escape the notice of the observer.

The results given in the table represent measurements of the variation of level of the lunar surface.

So long as the variations of the level or lunar radius are unknown, the limb is an uncertain index of the position of the Moon's centre in the sky, and lunar tables based on observations at the limb will require correction for the variation of radius.

In order to exhibit on a globe or map of the Moon the variation of level at different points on the lunar surface, the positions of the points can be expressed by Hayn's coordinates. The centre O of the map (which centre is the intersection of the first meridian and the equator) is taken as the pole of coordinates, and the arc O H drawn to the Moon's north pole is the initial line. If R be any point on the surface and P and D its coordinates, P is the position angle H O R of the polar distance O R, and D is $90° - O R$.

As the libration varies, the limb travels in a zone surrounding the map and extending to the parallels $D = \pm 8°$, or further, on either side of the equator of the system.

The variation of level in this occultation zone is measured, for the purpose of the present table, from the surface of a sphere described about the Moon's centre at a distance $r = 15' 32''{\cdot}50$.

This system has been used in the survey of levels published by Dr. Przybyllok (*Mittheilungen der Grossh. Sternwarte zu Heidelberg*, xi.). His table is based on results obtained by Hartwig, Hayn, and Battermann, the number of observations being 1032; these measures of the level are at points distributed over nearly the whole area of the occultation zone; the results given in the table below are confined to a narrow strip of surface, with the exception of some points observed from places remote from Europe, and displaced by parallax.

In the table below, the column headed "G.M.T." contains the Greenwich Mean Time used in the computation as the observed time. The values a, b, and c are the amounts by which the computed radius of the Moon would be increased if there were an increase of one second of arc in (1) the right ascension of the star, (2) its declination, (3) the assumed constant of parallax.

v is the computed vertical distance of the Moon's surface at R above or below the surface of the sphere of radius $r = 15' 32''{\cdot}50$.

v' is the corresponding value taken from Dr. Przybyllok's table.

Mean values of the discordance $v - v'$ over different portions of the limb are shown in a list added at the end of the table: in taking the mean the values of v in brackets were not included.

Catalogue of Stars.

No.	R.A. 1898·0.	Declination 1898·0.	Corrections applied, $\Delta\alpha$ (in Arc).	$\Delta\delta$.
	h m s	° ′ ″	″	″
16	6 23 59·14	23 47 50·2	−0·2	...
23	25 7·32	22 52 4·9	+0·1	+0·1
24	9·29	23 48 39·1	...	−0·1
25	12·52	24 19 24·7	+0·1	...
(118)	37·24	24 8 55·1	...	−0·1

Catalogue of Stars—continued.

No.	R.A. 1898·0.	Declination 1898·0.	Corrections applied, $\Delta\alpha$ (in Arc).	$\Delta\delta$.
	h m s	° ′ ″	″	″
26	6 25 38·17	22 51 40·3	−0·1	+0·1
(127)	55·38	24 25 2·3	+0·1	+0·1
27	26 1·20	23 52 50·5	+0·2	−0·2
28	19·69	22 55 46·7	+0·1	+0·2
29	20·77	22 54 35·2	...	−0·1
(136)	22·15	22 53 51·0	−0·1	−0·1
30	29·37	23 11 31·7	−0·2	+0·1
a	30·496	23 19 51·25	...	...
31	31·22	23 26 49·5	...	−0·1
b	33·459	24 23 47·60	...	...
32	33·62	23 9 32·6	−0·1	+0·2
(142)	36·55	22 56 0·2	...	+1·4
(143)	39·01	22 54 54·6	+0·1	−0·1
33	46·64	24 21 52·5	...	+0·2
34	47·46	23 20 38·9	+0·2	−0·2
(157)	59·08	23 2 21·6	−0·1	−0·1
36	27 0·98	23 11 5·9	−0·1	...
37	4·59	24 6 59·6	−0·2	−0·2
39	15·89	23 15 14·1	+0·1	...
c	16·120	22 49 35·33	...	...
41	18·72	23 28 48·9	+0·1	−0·1
d	19·436	24 6 56·47	...	...
e	22·670	22 50 15·70	...	...
42	33·73	24 8 25·8	−0·3	−0·1
43	35·54	23 27 34·6	+0·1	−0·1
44	35·54	24 14 46·0	+0·1	−0·1
45	35·80	23 8 0·5	−0·2	+0·1
46	50·73	23 7 32·5	...	+0·2
47	53·02	24 23 30·1	...	...
48	53·23	24 7 30·7	−0·2	−0·1
49	55·94	24 19 0·4	...	+0·1
(186)	56·44	24 15 20·1	+0·1	+0·3
50	57·86	22 51 39·7	+0·4	−0·2
f	58·884	22 53 31·41	...	...
g	28 7·050	23 15 49·38	...	...
h	15·399	23 19 16·04	...	...
53	24·79	23 16 43·1	−0·1	+0·1
56	33·64	24 13 37·5	...	...

Catalogue of Stars—continued.

No.	R.A. 1898·0.	Declination 1898·0.	Corrections applied, $\Delta\alpha$ (in Arc).	$\Delta\delta$.
	h m s	° ′ ″	″	″
58	6 28 38·38	23 27 39·5	−0·1	−0·1
64	57·09	22 56 23·2	−0·2	...
70	29 45·37	23 1 59·7	+0·2	...
71	46·87	23 1 1·9	+0·3	...
72	30 0·33	23 2 8·6	+0·2	...
74	26·22	24 15 30·8	...	...
75	31·34	23 10 52·4	−0·1	+0·2
76	35·45	22 44 11·8	−0·1	−0·1
78	40·92	23 4 57·2	−0·1	+0·2
79	52·22	22 48 27·8	−0·1	...
80	57·67	23 7 34·8	+0·1	...
81	31 3·17	24 6 14·5	...	...
k	15·932	22 47 0·77	...	...
83	24·65	23 58 0·3	...	...
84	28·81	22 48 8·1	+0·1	+0·1
87	35·73	23 15 45·9	+0·1	...
88	35·84	22 46 40·9	−0·2	−0·1
(305)	36·64	23 15 29·1	−0·2	...
89	37·64	23 11 56·0	−0·1	−0·1
92	55·78	23 11 21·4	+0·1	...
94	32 38·98	23 9 21·4	...	...
96	53·60	23 3 0·1	...	...

Table used for Reducing the Places of the Stars from 1898·0 *to* 1898 *Dec.* 27.

	R.A. 6h 24m	R.A. 6h 33m	Declination.
$\Delta\alpha$	+ 6ˢ·315	+ 6ˢ·312	22° 30′
$\Delta\delta$	− 2″·81	− 4″·16	

			Declination.
$\Delta\alpha$	+ 6ˢ·412	+ 6ˢ·407	24° 30′
$\Delta\delta$	− 2″·80	− 4″·18	

Table of Computed Values of the Vertical Distance v of the Lunar Surface above or below the Surface of a Sphere of Radius 15′ 32″·50.

No.	Place.	G.M.T.	Star.	*a*.	*b*.	*c*.	P.	D.	*v*.	*v′*.
		h m s		″	″	″	° ′	° ′	″	″
1	Flower Observatory, Pa.	12 5 46·49	87	+0·260	+1·015	+0·292	13 59	+0 36	−0·19	0·0
2	do.	12 4 9·27	(305)	+ ·321	+ ·995	+ ·237	17 45	+0 45	+0·26	+0·2
3	Liverpool	11 44 48·50	53	+ ·420	+ ·949	+ ·418	24 27	+0 46	−0·08	−0·2
4	Madrid	11 48 35·80	58	+ ·438	+ ·940	+ ·215	25 42	+0 58	−0·41	0·6
5	Prague	12 13 8·00	53	+ ·454	+ ·931	+ ·494	26 46	+0 43	−0·03	−0·8
6	Nicolaieff	11 40 47·30	34	+ ·462	+ ·926	+ ·523	27 27	+0 44	−0·19	−0·8
7	Leipzig	11 2 0·91	34	+ ·521	+ ·888	+ ·379	31 32	+1 0	−0·32	−0·8
8	Königsberg	11 32 0·95	39	+ ·540	+ ·874	+ ·510	32 51	+0 52	−0·37	−0·7
9	Rio de Janeiro	11 24 50·80	74	+ ·551	+ ·866	− ·897	33 14	+2 11	−1·00	0·6
10	Jena University	10 58 11·00	34	+ ·575	+ ·847	+ ·343	35 26	+1 7	−1·15	−0·6
11	Bamberg	12 0 52·60	53	+ ·596	+ ·830	+ ·426	36 47	+1 0	−0·09	−0·6
12	Pulkovo	11 9 49·30	30	+ ·630	+ ·799	+ ·557	39 40	+1 0	+0·29	−0·1
13	Madrid	10 41 46·60	41	+ 649	+ ·781	+ ·027	41 5	+1 17	−0·44	+0·2
14	Bamberg	10 52 28·82	34	+ ·670	+ ·759	+ ·272	42 50	+1 19	−0·33	+0·4
15	Heidelberg Astrophys. Abth.	11 53 38·56	53	+ ·674	+ ·756	+ ·365	42 59	+1 10	−0·84	+0·5
16	,, Astrometr. ,,	11 53 38·01	53	+ ·674	+ ·755	+ ·365	43 0	+1 10	−0·71	+0·5
17	Durban	11 31 8·00	*b*	+ ·698	+ ·729	− ·288	45 7	+1 49	−0·43	−0·2
18	Nicolaieff	11 18 50·40	*a*	+ ·705	+ ·721	+ ·468	45 44	+1 9	−0·04	−0·5
19	Madrid	10 50 50·40	43	+ ·705	+ ·720	+ ·008	45 45	+1 37	−0·24	−0·6

20	Princeton	12 1 11·40	92	+ ·716	+ ·708	− ·200	46 10	+1 42	−0·15	−0·7
21	Heidelberg, Astrometr. Abth.	10 46 29·34	34	+ ·715	+ ·709	+ ·213	46 34	+1 26	−0·61	−0·7
22	Munich	11 57 22·72	53	+ ·717	+ ·708	+ ·372	46 36	+1 12	−0 17	−0·7
23	Flower Observatory	12 0 16·68	92	+ ·720	+ ·703	− ·211	46 36	+1 44	−1·01	−0·7
24	Bothkamp	11 8 33·30	39	+ ·717	+ ·707	+ ·319	46 43	+1 19	−0·63	−0·8
25	Warsaw	11 26 2·55	39	+ ·725	+ ·697	+ ·419	47 29	+1 14	−0·21	−0·8
26	Strassburg	11 49 8·85	53	+ ·739	+ ·680	+ ·313	48 35	+1 18	−1·05	−0·8
27	Princeton	11 18 28·05	75	+ ·746	+ ·670	− ·225	49 5	+1 51	−1·03	−0·8
28	Hamburg	11 6 39·9	39	+ ·752	+ ·661	+ ·285	50 3	+1 24	−0·90	−0·6
29	Washington	11 16 5·4	75	+ ·760	+ ·651	− ·261	50 28	+1 54	−0·95	−0·6
30	Charlottesville, Va.	11 14 45·8	75	+ ·769	+ ·638	− ·284	51 19	+1 56	−0·98	−0·5
31	Pulkovo	11 6 55·7	32	+ ·766	+ ·643	+ ·484	51 23	+1 14	−0·60	−0·4
32	Durban	11 37 30·5	33	+ ·767	+ ·642	− ·180	51 26	+2 36	−1·58	−0·6
33	Stonyhurst	10 47 0·6	39	+ ·768	+ ·640	+ ·147	51 32	+1 35	−0·70	−0·5
34	Berlin	11 10 51·8	39	+ ·783	+ ·618	+ ·291	52 58	+1 26	−0·52	−0·5
35	Dublin	10 40 30·9	39	+ ·783	+ ·617	+ ·089	53 3	+1 40	−0·90	−0·4
36	Zürich	11 48 20·9	53	+ ·793	+ ·603	+ ·281	53 52	+1 24	−0·83	−0·5
37	San Fernando	11 23 33·9	58	+ ·797	+ ·597	− ·052	54 17	+1 45	−0·85	−0·5
38	Cape of Good Hope	11 52 8·7	47	+ ·797	+ ·597	− 285	54 19	+1 55	−0·98	−0·5
39	Princeton	12 20 20·8	94	+ ·811	+ ·574	− ·337	55 24	+1 56	−0·95	−0·5
40	Durban	10 56 45	(127)	+ ·805	+ ·583	− ·252	55 28	+1 57	−0·94	−0·5
41	Nicolaieff	11 43 47·3	39	+ ·816	+ ·566	+ ·497	56 27	+1 15	+0·06	−0·5

Table of Computed Values of the Vertical Distance v of the Lunar Surface above or below the Surface of a Sphere of Radius 15′ 32″·50—continued.

No.	Place.	G.M.T.	Star.	*a*.	*b*.	*c*.	P.	D.	*v*.	*v′*.
		h m s		″	″	″	° ′	° ′	″	″
42	Besançon	11 42 9·8	53	+0·819	+0·560	+0·223	56 41	+1 30	−0·09	−0·5
43	Leipzig	11 6 48·59	39	+ ·836	+ ·530	+ ·223	58 45	+1 34	−0·49	−0·4
44	Odessa	11 40 47·9	39	+ ·848	+ ·506	+ ·462	60 15	+1 20	−0·67	−0·4
45	Jena (W. Winkler)	11 4 37·0	39	+ ·853	+ ·496	+ ·192	60 51	+1 37	−0·20	−0·4
46	Jena University	11 4 36·4	39	+ ·853	+ ·495	+ ·191	60 53	+1 37	−0·58	−0·4
47	Geneva	11 40 48·0	53	+ ·856	+ ·489	+ ·186	61 10	+1 35	−0·69	−0·4
48	Prague	11 8 57·39	39	+ ·867	+ ·465	+ ·211	62 42	+1 36	−0·32	−0·4
49	Königsberg	11 8 15·5	36	+ ·874	+ ·450	+ ·289	63 40	+1 33	−0·68	−0·4
50	Princeton	11 26 51·1	80	+ ·880	+ ·436	− ·458	64 5	+2 11	−1·21	−0·4
51	Lyons	11 37 1·2	53	+ ·879	+ ·439	+ ·140	64 11	+1 39	−1·25	−0·4
52	Turin	11 42 47·42	53	+ ·883	+ ·430	+ ·181	64 45	+1 36	−0·98	−0·4
53	Bamberg	11 1 44·90	39	+ ·886	+ ·420	+ ·135	65 26	+1 42	−0·36	−0·4
54	Frankenthal	10 56 27·70	39	+ ·902	+ ·378	+ ·075	67 47	+1 47	−0·76	−0·3
55	Heidelberg, Astrophys. Abth.	10 56 59·46	39	+ ·904	+ ·373	+ ·076	68 10	+1 47	−0·98	−0·3
56	,, Astrometr. ,,	10 56 58·46	39	+ ·904	+ ·373	+ ·076	68 11	+1 47	−0·65	−0·3
57	Königsberg	11 38 33·50	46	+ ·905	+ ·370	+ ·323	68 19	+1 30	−0·34	−0·3
58	,,	10 46 41·6	30	+ ·905	+ ·369	+ ·201	68 31	+1 40	−0·42	−0·3
59	Durban	12 6 5·	44	+ ·909	+ ·360	+ ·177	68 55	+1 37	−0·45	−0·3
60	Königsberg	11 28 43·45	45	+ ·913	+ ·345	+ ·288	69 45	+1 33	+0·06	−0·3

61	Bordeaux	11	24	15·1	53	+ ·916	+ ·337	− ·002	70 8	+1 50	+0·18	−0·3
62	Strassburg	10	54	10·1	39	+ ·922	+ ·314	+ ·028	71 34	+1 50	−0·99	−0·3
63	Munich	11	1	11·21	39	+ ·923	+ ·311	+ ·085	71 44	+1 47	−0·83	−0·3
64	Warsaw	11	7	33·7	36	+ ·926	+ ·301	+ ·216	72 15	+1 40	−0·74	−0·3
65	Toulouse	11	27	7·15	53	+ ·935	+ ·267	− ·003	74 6	+1 50	−0·57	−0·3
66	Cape of Good Hope	11	47	4·0	49	+ ·940	+ ·245	+ ·010	75 23	+1 48	−0·22	−0·2
67	Zürich	10	54	22·7	39	+ ·941	+ ·239	− ·002	75 48	+1 53	−0·41	−0·2
68	Königsberg	10	47	47·5	32	+ ·942	+ ·234	+ ·137	76 9	+1 45	+0·58	−0·2
69	Warsaw	11	39	5·8	46	+ ·943	+ ·232	+ ·266	76 10	+1 34	+0·55	−0·2
70	Cape of Good Hope	10	44	7·4	*b*	+ ·943	+ ·230	− ·195	76 18	+2 4	+0·14	−0·2
71	Bothkamp	11	21	6·2	46	+ ·944	+ ·223	+ ·108	76 32	+1 44	+0·11	−0·2
72	Charlottesville	11	13	44·3	78	+ ·946	+ ·216	− ·656	76 44	+2 26	−0·14	−0·2
73	Bordeaux	11	10	35·8	*g*	+ ·945	+ ·218	− ·085	76 47	+1 56	−0·04	−0·2
74	Warsaw	10	46	0·3	30	+ ·945	+ ·220	+ ·124	76 56	+1 46	+0·19	−0·2
75	Besançon	10	49	16·3	39	+ ·945	+ ·218	− ·055	76 57	+1 56	−0·18	−0·2
76	Rio de Janeiro	11	31	21·5	81	+ ·947	+ ·208	− ·825	77 8	+2 35	−0·84	−0·2
77	Warsaw	11	29	13·7	45	+ ·948	+ ·206	+ ·230	77 36	+1 38	+0·08	−0·2
78	Madrid	11	7	18·9	*h*	+ ·951	+ ·187	− ·172	78 30	+2 2	−0·68	−0·2
79	Leipzig	10	51	44·02	36	+ ·954	+ ·168	+ ·021	79 47	+1 53	−0·76	0·0
80	Nicolaieff	11	27	53·4	36	+ ·955	+ ·165	+ ·337	79 57	+1 33	−0·02	0·0
81	Berlin	11	25	57·3	46	+ ·956	+ ·153	+ ·117	80 32	+1 44	+0·78	+0·1
82	Geneva	10	48	36·1	39	+ ·956	+ ·151	− ·087	80 42	+1 58	+0·38	+0·1

Table of Computed Values of the Vertical Distance v of the Lunar Surface above or below the Surface of a Sphere of Radius 15′ 32″·50—continued.

No.	Place.	G.M.T.	Star.	*a*.	*b*.	*c*.	P.	D.	*v*.	*v′*.
		h m s		″	″	″	° ′	° ′	″	″
83	Stonyhurst	11 0 57·5	46	+0·957	+0·147	−0·107	80 51	+1 57	+0·54	+0·1
84	,,	10 51 32·2	45	+ ·957	+ ·146	− ·129	80 56	+2 0	−0·23	+0·1
85	Nicolaieff	12 0 9·3	46	+ ·958	+ ·140	+ ·414	81 13	+1 26	+0·43	+0·2
86	Cape of Good Hope	10 53 28·3	33	+ ·957	+ ·142	− ·092	81 14	+2 58	−0·05	−0·1
87	Jena University	10 50 0·3	36	+ ·958	+ ·137	− ·008	81 29	+1 54	+0·78	+0·1
88	Flower Observatory	12 21 55·46	96	+ ·961	+ ·116	− ·670	82 4	+2 23	−0·03	+0·1
89	Dublin	10 45 29·5	45	+ ·960	+ ·121	− ·196	82 12	+2 4	+0·32	+0·1
90	Liverpool	10 59 37·4	46	+ ·960	+ ·120	− ·132	82 21	+1 59	+0·70	+0·1
91	Lyons	10 45 30·8	39	+ ·961	+ ·113	− ·130	82 45	+2 0	+0·61	+0·2
92	Leipzig	10 30 35·64	30	+ ·961	+ ·107	− ·062	83 10	+1 57	−0·17	+0·2
93	Washington	12 19 35·9	96	+ ·963	+ ·094	− ·704	83 16	+2 25	−0·09	+0·2
94	Nicolaieff	11 50 21·4	45	+ ·962	+ ·102	+ ·374	83 18	+1 28	+0·04	+0·3
95	Warsaw	10 47 59·3	32	+ ·963	+ ·090	+ ·071	84 4	+1 49	+0·78	+0·2
96	Odessa	11 58 28·6	46	+ ·964	+ ·087	+ ·379	84 7	+1 28	+0·86	+0·3
97	Turin	10 50 52·53	39	+ ·963	+ ·082	− ·094	84 20	+1 57	+0·79	+0·2
98	Leipzig	11 24 4·89	46	+ ·964	+ ·073	+ ·065	84 54	+1 47	+0·68	+0·3
99	,,	11 14 12·37	45	+ ·965	+ ·056	+ ·030	85 50	+1 49	−0·12	+0·3
100	Bordeaux	10 34 42·2	39	+ ·965	+ ·054	− ·249	85 58	+2 7	+0·76	+0·3
101	Nicolaieff	11 6 21·6	30	+ ·965	+ ·056	+ ·237	85 59	+1 38	+0·33	+0·3

102	Odessa	11 48 40·7	45	+ ·966	+ ·050	+ ·338	86 8	+1 30	+0·04	+0·4
103	Liverpool	12 13 14·0	70	+ ·966	+ ·046	+ ·014	86 9	+1 47	+0·63	+0·3
104	Cape of Good Hope	12 13 47·35	56	+ ·966	+ ·044	+ ·267	86 23	+1 31	+0·18	+0·3
105	Jena (W. Winkler)	11 22 40·4	46	+ ·966	+ ·042	+ ·037	86 36	+1 48	+0·46	+0·4
106	Jena University	11 22 40·1	46	+ ·966	+ ·041	+ ·037	86 38	+1 48	+0·28	+0·4
107	Madrid	11 13 14·8	53	+ ·966	+ ·030	− ·202	87 11	+2 2	−0·29	+0·4
108	Heidelberg, Astrometr. Abth.	10 44 12·27	36	+ ·966	+ ·028	− ·109	87 26	+1 59	−0·26	+0·4
109	Jena University	11 12 44·6	45	+ ·966	+ ·025	+ ·003	87 33	+1 50	+0·16	+0·4
110	Prague	11 27 44·12	46	+ ·966	+ ·021	+ ·074	87 45	+1 45	+0·67	+0·4
111	Charlottesville	10 47 54·2	70	+ ·966	+ ·010	− ·778	88 7	+2 34	+0·61	+0·3
112	Odessa	11 4 22·8	30	+ ·966	+ ·006	+ ·200	88 40	+1 40	+0·21	+0·5
113	Prague	11 17 46·22	45	+ ·966	+ ·001	+ ·039	88 50	+1 48	+0·38	+0·4
114	Cape of Good Hope	11 46 6·7	(186)	+ ·967	· ·021	+ ·225	89 59	+1 36	+0·27	+0·5
115	Leipzig	10 33 10·05	32	+ ·966	− ·022	− ·114	90 12	+1 58	+0·19	+0·4
116	Strassburg	10 42 8·5	36	+ ·966	− ·027	− ·150	90 23	+2 0	+0·13	+0·4
117	Bamberg	11 21 27·87	46	+ ·966	− ·028	− ·019	90 23	+1 49	+0·43	+0·4
118	Christiania	12 1 24·6	64	+ ·966	− ·032	+ ·072	90 23	+1 43	+0·54	+0·5
119	Dublin	12 8 56·9	71	+ ·966	− ·041	− ·073	90 53	+1 50	+0·57	+0·4
120	Bamberg	11 11 29·84	45	+ ·965	− ·044	− ·037	91 15	+1 52	+0·23	+0·4
121	Durban	10 14 57	25	+ ·964	− ·061	+ ·120	92 25	+1 46	−0·04	+0·4
122	,,	12 15 56·5	48	+ ·965	− ·074	+ ·543	92 52	+1 15	+0·48	+0·4
123	Madrid	11 0 45·7	*g*	+ ·964	− ·078	− ·272	92 58	+2 5	+0·19	+0·3

Table of Computed Values of the Vertical Distance v of the Lunar Surface above or below the Surface of a Sphere of Radius 15′ 32″·50—continued.

No.	Place.	G.M.T.	Star.	*a*.	*b*.	*c*.	P.	D.	*v*.	*v′*.
		h m s		″	″	″	° ′	° ′	″	″
124	Heidelberg, Astrometr. Abth.	11 17 37·96	46	+0·964	−0·075	−0·059	92 58	+1 51	+0·09	+0·3
125	,, Astrophys. ,,	11 17 37·56	46	+ ·964	− ·075	− ·059	92 58	+1 51	+0·19	+0·3
126	Heidelberg, Astrometr. Abth.	11 7 39·36	45	+ ·963	− ·089	− ·092	93 43	+1 54	−0·30	+0·2
127	Odessa	11 7 29·3	32	+ ·961	− ·114	+ ·162	95 12	+1 39	+0·94	+0·2
128	Durban	12 2 16	42	+ ·961	− ·120	+ ·533	95 26	+1 16	+0·66	+0·2
129	Munich	11 23 8·93	46	+ ·960	− ·124	− ·028	95 38	+1 48	+0·98	+0·2
130	Strassburg	11 16 0·7	46	+ ·959	− ·132	− ·096	96 2	+1 52	+0·17	+0·2
131	Munich	11 13 7·17	45	+ ·958	− ·141	− ·064	96 35	+1 51	+0·13	+0·2
132	Strassburg	11 5 58 9	45	+ ·957	− ·144	− ·130	96 45	+1 55	+0·12	+0·2
133	Cape of Good Hope	11 30 54·8	44	+ ·956	− ·156	+ ·277	97 22	+1 30	−0·32	+0·1
134	Königsberg	12 18 7·85	64	+ ·954	− ·171	+ ·147	98 4	+1 28	−0·04	−0·1
135	,,	11 3 45·15	(157)	+ ·950	− ·190	− ·033	99 20	+1 48	+0·20	−0·3
136	Zürich	11 17 47·9	46	+ ·949	− ·198	− ·105	99 40	+1 50	+0·29	−0·4
137	,,	11 7 41·1	45	+ ·947	− ·211	− ·140	100 27	+1 53	+0·48	−0·5
138	Madrid	10 26 6·9	39	+ ·945	− ·218	− ·414	100 51	+2 9	−0·12	−0·5
139	Besançon	11 13 3·6	46	+ ·944	− ·224	− ·159	101 8	+1 53	+0·19	−0·6
140	San Fernando	11 6 26·8	53	+ ·943	− ·230	− ·327	101 24	+2 2	+0·34	−0·6
141	Besançon	11 2 58·1	45	+ ·942	− ·235	− ·192	101 45	+1 55	+0·06	−0·7
142	San Fernando	12 40 42·65	75	+ ·941	− ·245	− ·013	101 56	+1 41	+0·77	−1·0

143	Geneva	11 13 49·1	46	+ ·930	− ·286	− ·174	104 35	+1 51	−0·06	−1·0
144	Durban	11 53 38·0	*d*	+ ·928	− ·297	+ ·619	105 9	+1 7	−1·00	−1·3
145	Geneva	11 3 40·8	45	+ ·927	− ·297	− ·207	105 14	+1 53	−0·50	−1·1
146	Lyons	11 11 31·45	46	+ ·919	− ·325	− ·209	106 48	+1 52	−1·16	−1·2
147	,,	11 1 18·7	45	+ ·916	− ·334	− ·242	107 21	+1 53	−0·55	−1·2
148	Pulkovo	11 2 38·9	(142)	+ ·915	− ·339	− ·065	107 40	+1 44	−0·10	−1·3
149	Mauritius	11 58 1·3	27	+ ·912	− ·349	+ ·916	108 16	+0 48	−2·09	−1·4
150	Rio de Janeiro	11 47 21·7	83	+ ·904	− ·373	− ·424	109 15	+2 0	(−3·71)	−1·1
151	Durban	11 43 53	37	+ ·906	− ·367	+ ·632	109 17	+1 3	−1·45	−1·4
152	Berlin	12 11 27·2	64	+ ·903	− ·376	− ·026	109 39	+1 31	+0·16	−1·2
153	Bordeaux	11 1 44·5	46	+ ·895	− ·397	− ·320	110 59	+1 54	−0·80	−1·0
154	,,	10 51 35·2	45	+ ·894	− ·400	− ·349	111 12	+1 57	−1·37	−1·0
155	Pulkovo	10 53 26·9	28	+ ·892	− ·406	− ·123	111 40	+1 44	−0·24	−1·0
156	Leipzig	12 11 41·88	64	+ ·876	− ·447	− ·061	113 51	+1 29	−0·01	−0·6
157	Toulouse	11 6 25·35	46	+ ·867	− ·465	− ·297	115 0	+1 49	−0·39	−0·3
158	,,	10 56 7·32	45	+ ·865	− ·470	− ·327	115 22	+1 51	−0·20	−0·2
159	Liverpool	11 46 30·70	64	+ ·862	− ·476	− ·296	115 36	+1 47	−0·58	−0·2
160	Prague	12 16 29·64	64	+ ·860	− ·482	− ·038	115 56	+1 26	−0·45	−0·1
161	Pulkovo	10 55 22·1	29	+ ·860	− ·479	− ·165	116 4	+1 42	−0·14	−0·1
162	Washburn Observatory	11 45 59·75	84	+ ·837	− ·527	− ·958	118 28	+2 20	+0·44	+0·6
163	Pulkovo	10 57 1·6	(136)	+ ·839	− ·523	− ·191	118 40	+1 40	+0·53	+0·6
164	Cape of Good Hope	11 50 45·65	48	+ ·827	− ·546	+ ·623	120 0	+0 53	+0·98	+0·4

Table of Computed Values of the Vertical Distance v of the Lunar Surface above or below the Surface of a Sphere of Radius 15′ 32″·50—continued.

No.	Place.	G.M.T.	Star.	*a*.	*b*.	*c*.	P.	D.	*v*.	*v*′.
		h m s		″	″	″	° ′	° ′	″	″
165	Uccle	12 0 45·5	64	+0·818	−0·563	−0·213	120 59	+1 31	+0·39	+0·4
166	Königsberg	11 47 33·9	*f*	+ ·815	− ·567	− ·146	121 18	+1 34	−0·47	+0·3
167	Cape of Good Hope	11 36 49·4	42	+ ·814	− ·569	+ ·601	121 32	+0 54	−0·04	+0·1
168	Frankenthal	12 8 50·1	64	+ ·803	− ·588	− ·158	122 33	+1 33	−1·09	+0·3
169	Heidelberg, Astrometr. Abth.	12 9 32·96	64	+ ·802	− ·589	− ·152	122 40	+1 32	+0·04	+0·3
170	,, Astrophys. ,,	12 9 33·56	64	+ ·802	− ·589	− ·152	122 41	+1 32	−0·20	+0·3
171	Washburn Observatory	11 41 35·1	*k*	+ ·789	− ·609	− ·973	123 41	+2 16	+2·14	+0·5
172	Munich	12 15 59·66	64	+ ·786	− ·614	− ·112	124 20	+1 22	+1·07	+0·7
173	Strassburg	12 9 36·50	64	+ ·768	− ·641	− ·178	126 6	+1 30	+0·94	+1·5
174	Madrid	10 48 53·1	45	+ ·751	− ·663	− ·421	127 50	+1 41	+1·32	+1·0
175	,,	10 59 22·5	46	+ ·751	− ·663	− ·393	127 50	+1 40	+1·21	+1·0
176	Königsberg	10 58 1·5	(142)	+ ·748	− ·667	− ·295	128 14	+1 35	+1·06	+0·9
177	,,	10 50 10·15	50	+ ·738	− ·681	− ·213	129 3	+1 28	+1·02	+0·7
178	Madrid	12 23 26·7	70	+ ·722	− ·701	− ·184	130 18	+1 26	+1·07	+0·2
179	Cape of Good Hope	11 31 3·2	*d*	+ ·714	− ·711	+ ·683	131 13	+0 37	+0·46	+0·1
180	Königsberg	11 1 44·1	(143)	+ ·700	− ·727	− ·320	132 32	+1 30	+0·03	−0·1
181	,,	10 49 25·6	28	+ ·699	− ·728	− ·342	132 41	+1 31	+0·29	−0·1
182	Cape of Good Hope	11 22 2·6	37	+ ·671	− ·759	+ ·694	134 58	+0 30	+0·48	−0·1
183	Geneva	12 12 27·55	64	+ ·657	− ·774	− ·219	135 57	+1 14	+0·70	−0·2

184	Königsberg	10 52 54·2	29	+ ·634	− ·795	− ·372	137 57	+1 29	−0·12	−0·2
185	Turin	12 17 39·78	64	+ ·620	− ·809	− ·202	138 51	+1 9	+0·18	−0·2
186	Washburn Observatory	11 28 41·1	76	+ ·614	− ·814	·977	139 8	+1 56	−0·44	−0·2
187	Lyons	12 11 48·5	64	+ ·616	− ·813	− ·244	139 10	+1 12	+0·09	−0·2
188	Nicolaieff	12 20 19·2	50	+ ·594	− ·832	− ·111	140 54	+1 8	−0·64	−0·2
189	Königsberg	10 55 38·85	(136)	+ ·591	− ·834	− ·389	141 14	+1 21	−0·80	−0·2
190	Mauritius	11 39 1·3	24	+ ·589	− ·836	+ ·885	141 29	+0 12	(+0·89)	−0·3
191	Warsaw	10 55 43·0	28	+ ·582	− ·841	− ·357	141 55	+1 18	−0·39	−0·2
192	Königsberg	11 35 53·4	*c*	+ ·550	− ·867	− ·357	144 5	+1 15	−0·42	−0·2
193	Nicolaieff	11 30 37·7	(142)	+ ·531	− ·881	− ·203	145 38	+1 5	−0·62	−0·1
194	San Fernando	10 51 43·2	45	+ ·521	− ·887	− ·389	146 10	+1 14	−0·15	−0·1
195	,,	11 2 32·0	46	+ ·519	− ·889	− ·366	146 18	+1 12	−0·04	−0·1
196	Warsaw	11 0 0·2	29	+ ·500	− ·902	− ·384	147 47	+1 16	+0·90	0·0
197	Leipzig	11 49 52·08	50	+ ·489	− ·909	− ·361	148 20	+1 10	+0·28	+0·1
198	Königsberg	11 34 56·4	*c*	+ ·475	− ·918	− ·396	149 24	+1 10	+0·79	+0·4
199	Nicolaieff	11 35 15·4	(143)	+ ·462	− ·926	− ·238	150 22	+1 0	+1·72	+0·7
200	Leipzig	10 46 52·92	28	+ ·450	− ·933	− ·451	151 14	+1 8	+1·36	+0 8
201	Princeton	11 56 54·65	84	+ ·438	− ·940	− ·807	151 31	+1 30	+1·30	+0·8
202	Warsaw	11 3 19·2	(136)	+ ·444	− ·936	− ·401	151 32	+1 6	+1·44	+0·8
203	Toulouse	12 13 34·15	64	+ ·440	− ·939	− ·289	151 36	+0 57	+1·47	+0·8
204	Prague	11 56 2·85	50	+ ·439	− ·939	− ·346	151 45	+1 4	+1·75	+0·8
205	Nicolaieff	11 23 43·7	28	+ ·435	− ·942	·261	152 15	+0 57	+1·91	+0·8

Table of Computed Values of the Vertical Distance v of the Lunar Surface above or below the Surface of a Sphere of Radius 15′ 32″·50—continued.

No.	Place.	G.M.T.	Star.	*a*.	*b*.	*c*.	P.	D.	*v*.	*v′*.
		h m s		″	″	″	° ′	° ′	″	″
206	Flower Observatory	11 56 36·99	84	+0·423	−0·948	−0·801	152 28	+1 28	+1·89	+0·9
207	Jena University	10 47 10·8	28	+ ·410	− ·955	− ·456	153 52	+1 4	+1·11	+1·0
208	Charlottesville	11 36 9·1	79	+ ·402	− ·959	− ·820	153 56	+1 26	+1·40	+1·0
209	Washington	11 55 56·75	84	+ ·388	− ·966	− ·786	154 47	+1 23	+1·52	+1·1
210	Bamberg	11 53 25·52	50	+ ·362	− ·977	− ·384	156 47	+0 58	+1·61	+1·0
211	Leipzig	10 52 50·23	29	+ ·331	− ·990	− ·462	158 53	+0 58	+1·79	+0·8
212	Nicolaieff	11 29 3·1	29	+ ·331	− ·990	− ·306	158 55	+0 50	+1·96	+0·8
213	Bamberg	10 50 35·42	28	+ ·301	−1·00	− ·453	160 49	+0 53	+0·03	+0·4
214	Nicolaieff	12 15 2·20	(136)	+ ·257	−1·02	− ·338	163 29	+0 43	+0·09	−0·3
215	Odessa	11 30 55·0	29	+ ·251	−1·02	− ·334	163 53	+0 43	+0·73	−0·4
216	Heidelberg, Astrometr. Abth.	10 50 17·71	28	+ ·218	−1·03	− ·457	165 55	+0 44	+0·32	−0·4
217	Munich	12 2 18·95	50	+ ·208	−1·03	− ·390	166 20	+0 43	+1·11	−0·4
218	Königsberg	10 45 43·4	26	+ ·167	−1·04	− ·514	169 1	+0 42	−1·08	−0·8
219	Strassburg	11 57 50·1	50	+ ·148	−1·04	− ·416	169 58	+0 38	−1·26	−1·0
220	Flower Observatory	12 10 57·17	88	+ ·109	−1·05	− ·588	171 54	+0 44	+0·95	−1·3
221	Bamberg	10 59 35·40	29	+ ·116	−1·05	− ·454	172 1	+0 34	−0·28	−1·3
222	Strassburg	10 55 35·28	28	+ ·056	−1·05	− ·439	175 35	+0 27	−0·34	−1·4
223	Munich	11 2 51·32	28	+ ·052	−1·05	− ·425	176 50	+0 24	+0·20	−1·2
224	Zürich	12 8 11·4	50	− ·040	−1·05	− ·420	181 5	+0 21	+0·15	−0·4

225	Mauritius	11 23 8·6	16	− ·089	−1·05	+ ·495	184 27	−0 37	−0·56	+0·1
226	Bamberg	11 10 27·89	(136)	− ·106	−1·05	− ·446	185 10	+0 12	−0·31	+0·6
227	Heidelberg, Astrometr. Abth.	11 13 16·25	29	− ·276	−1·01	− ·415	195 30	−0 6	−0·13	+1·4
228	Washington	12 22 44·3	84	− ·397	− ·961	− ·138	202 38	−0 26	+0·35	+1·0
229	Munich	11 24 35·23	28	− ·423	− ·948	− ·394	204 53	−0 18	−0·68	+1·0
230	Bamberg	11 26 8·26	29	− ·474	− ·919	− ·406	208 17	−0 23	−0·02	+0·9
231	Königsberg	11 15 37·9	26	− ·530	− ·881	− ·494	212 15	−0 25	−0·17	+0·8
232	Strassburg	12 30 15·7	50	− ·570	− ·852	− ·455	214 52	−0 23	+0·62	+0·5
233	Bamberg	11 32 16·65	28	− ·626	− ·803	− ·371	219 18	−0 37	+0·08	+0·1
234	Leipzig	11 36 38·67	29	− ·655	− ·775	− ·396	221 35	−0 39	+0·68	0·0
235	Berlin	11 40 30·3	(136)	− ·692	− ·736	− ·409	224 39	−0 41	−0·04	−0·3
236	Odessa	12 12 38·3	29	− ·702	− ·724	− ·583	225 31	−0 29	+0·01	−0·4
237	Nicolaieff	12 15 2·20	(136)	− ·709	− ·717	− ·602	226 2	−0 29	−0·15	−0·4
238	Berlin	11 41 44·2	29	− ·741	− ·677	− ·388	228 58	−0 46	−0·08	−0·6
239	Leipzig	11 39 46·69	28	− ·743	− ·675	− ·364	229 7	−0 47	−0·39	−0·6
240	San Fernando	11 54 20·7	45	− ·747	− ·669	− ·059	229 22	−1 3	+0·06	−0·6
241	Nicolaieff	12 16 10·90	29	− ·757	− ·657	− ·598	230 24	−0 33	−0·40	−0·6
242	San Fernando	12 6 12·1	46	− ·761	− ·651	− ·084	230 40	−1 7	−0·63	−0·6
243	,,	11 7 57·6	30	− ·799	− ·593	+ ·084	234 42	−1 18	−0·11	−0·6
244	Pulkovo	11 19 55·1	23	− ·843	− ·515	− ·453	239 47	−0 51	−1·16	−0·1
245	Königsberg	11 57 22·5	(136)	− ·859	− ·484	− ·420	241 35	−0 51	−0·20	−0·1
246	Charlottesville	11 22 29·4	64	− ·896	− ·396	+ ·522	246 34	−1 48	+0·10	−0·2

Table of Computed Values of the Vertical Distance v of the Lunar Surface above or below the Surface of a Sphere of Radius 15′ 32″·50—continued.

No.	Place.	G.M.T.	Star.	*a*.	*b*.	*c*.	P.	D.	*v*.	*v′*.
		h m s		″	″	″	° ′	° ′	″	″
247	Washington	11 23 48·6	64	−0·900	−0·383	+0·523	247 21	−1 47	+0·30	−0·2
248	Madrid	12 9 18·1	45	−·912	−·350	−·075	249 25	−1 13	+1·11	+0·1
249	Königsberg	11 58 12·8	28	−·914	−·342	−·361	249 59	−0 58	+0·61	+0·2
250	Madrid	12 20 30·7	46	−·919	−·327	−·103	250 42	−1 11	+0·73	+0·3
251	,,	11 45 26·7	36	−·941	−·241	+·031	255 38	−1 21	+1·11	+0·6
252	Toulouse	11 38 22·42	32	−·951	−·189	−·010	258 33	−1 19	+0·92	+0·6
253	San Fernando	11 49 53·8	39	−·953	−·179	+·087	259 3	−1 24	+0·89	+0·6
254	Christiania	11 44 8·6	28	−·954	−·168	−·153	259 44	−1 12	+1·44	+0·6
255	Bordeaux	11 34 54·04	32	−·960	−·122	+·043	262 13	−1 23	+1·26	+0·6
256	Toulouse	12 22 55·50	45	−·965	−·071	−·109	264 55	−1 11	−0·56	+0·2
257	,,	11 35 59·0	30	−·965	−·060	+·040	265 37	−1 22	−0·15	+0·1
258	,,	12 33 33·3	46	−·966	−·043	−·132	266 24	−1 10	+0·07	−0·2
259	Turin	11 52 2·87	32	−·966	−·029	−·078	267 20	−1 14	−0·90	−0·3
260	Lyons	11 46 1·8	32	−·966	−·024	−·022	267 36	−1 18	−1·17	−0·3
261	Durban	12 11 44·0	(118)	−·967	−·008	−·480	268 32	−0 51	−0·69	−0·7
262	Bordeaux	12 29 6·4	46	−·967	+·003	−·077	268 56	−1 12	−0·34	−0·6
263	,,	11 32 9·9	30	−·967	+·004	+·096	269 7	−1 24	−0·47	−0·2
264	Washington	11 49 24·6	70	−·967	+·016	+·778	269 27	−2 1	+0·41	−0·4
265	Toulouse	11 58 23·92	36	−·967	+·017	−·005	269 45	−1 18	−0·64	−0·7

266	Geneva	11 48 45·0	32	− ·967	+ ·022	− ·030	270 6	− 1 16	− 0·84	− 0·7
267	Besançon	11 48 10·6	32	− ·964	+ ·084	− ·001	273 28	− 1 16	− 1·06	− 0·8
268	Turin	11 48 56·31	30	− ·963	+ ·095	− ·026	274 4	− 1 15	− 0·36	− 0·7
269	Madrid	11 57 13·2	39	− ·962	+ ·102	+ ·092	274 23	− 1 22	− 0·53	− 0·7
270	Lyons	12 28 52·7	45	− ·962	+ ·103	− ·102	274 25	− 1 9	− 1·55	− 0·7
271	Cape of Good Hope	11 20 31·25	25	− ·961	+ ·112	− ·216	275 5	− 1 7	− 0·54	− 0·7
272	Zürich	11 52 59·4	32	− ·960	+ ·121	− ·031	275 30	− 1 14	− 0·16	− 0·6
273	Lyons	12 39 2·8	46	− ·960	+ ·132	− ·120	275 57	− 1 8	− 0·80	− 0·5
274	Geneva	12 31 5·6	45	− ·957	+ ·153	− ·104	277 8	− 1 8	− 0·57	− 0·4
275	Lyons	12 4 24·6	36	− ·952	+ ·182	− ·001	278 48	− 1 14	− 0·28	− 0·4
276	Strassburg	11 50 52·6	32	− ·952	+ ·182	+ ·014	278 50	− 1 15	0·15	− 0·4
277	Turin	12 10 27·18	36	− ·952	+ ·184	− ·056	278 54	− 1 10	+ 0·06	− 0·4
278	Besançon	12 29 52·9	45	− ·948	+ ·210	− ·069	280 15	− 1 9	− 0·47	− 0·4
279	Washington	12 16 46·0	78	− ·944	+ ·231	+ ·840	281 7	− 1 58	− 0·25	− 0·4
280	Heidelberg, Astrometr. Abth.	11 51 57·95	32	− ·942	+ ·239	+ ·028	282 0	− 1 13	− 0·15	− 0·4
281	,, Astrophys. ,,	11 51 58·56	32	− ·942	+ ·239	+ ·028	282 0	− 1 13	+ 0·11	− 0·4
282	Zürich	11 49 11·1	30	− ·941	+ ·241	+ ·028	282 6	− 1 14	− 0·62	− 0·4
283	Flower Observatory	12 18 50·96	78	− ·939	+ ·251	+ ·827	282 16	− 1 56	− 0·46	− 0·4
284	Zürich	12 34 7·9	45	− ·938	+ ·255	− ·088	282 48	− 1 5	− 0·13	− 0·4
285	,,	12 43 50·0	46	− ·931	+ ·284	− ·099	284 25	− 1 4	+ 0·03	0·0
286	Besançon	12 5 25·8	36	− ·931	+ ·286	+ ·030	284 36	− 1 13	+ 0·08	+ 0·1
287	Bamberg	11 55 8·11	32	− ·929	+ ·291	+ ·023	284 55	− 1 12	− 0·32	+ 0·2

Table of Computed Values of the Vertical Distance v of the Lunar Surface above or below the Surface of a Sphere of Radius 15′ 32″·50—continued.

No.	Place.	G.M.T.	Star.	*a*.	*b*.	*c*.	P.	D.	*v*.	*v′*.
		h m s		″	″	″	° ′	° ′	″	″
288	Strassburg	11 46 51·4	30	−0·927	+0·301	+0·074	285 29	−1 14	+0·29	+0·3
289	,,	12 31 28·2	45	·925	+ ·308	− ·039	285 45	−1 7	+0·53	+0·3
290	Munich	11 53 50·09	30	− ·921	+ ·321	+ ·019	286 37	−1 10	−0·23	+0·4
291	,,	12 37 58·35	45	− ·914	+ ·344	− ·081	287 48	−1 3	+0·07	+0·4
292	Prague	12 0 28·11	32	− ·913	+ ·347	− ·001	288 5	−1 9	−0·14	+0·4
293	Toulouse	12 6 24·57	39	− ·912	+ ·349	+ ·082	288 10	−1 12	+0·09	+0·4
294	Jena University	11 55 8·2	32	− ·910	+ ·355	+ ·052	288 35	−1 11	+0·07	+0·4
295	Jena (W. Winkler)	11 55 8·3	32	− ·910	+ ·356	+ ·052	288 37	−1 11	+0·16	+0·4
296	Heidelberg, Astrophys. Abth.	11 47 43·56	30	− ·910	+ ·356	+ ·090	288 38	−1 13	+0·74	+0·4
297	,, Astrometr. ,,	11 47 43·25	30	− ·910	+ ·356	+ ·089	288 39	−1 13	+0·62	+0·4
298	,, ,, ,,	12 31 53·95	45	− ·908	+ ·364	− ·017	288 58	−1 5	+0·43	+0·4
299	Strassburg	12 7 4·2	36	− ·902	+ ·381	+ ·056	290 2	−1 10	+0·46	+0·4
300	Madrid	11 33 42·5	34	− ·901	+ ·383	+ ·237	290 9	−1 20	+0·36	+0·4
301	Leipzig	11 55 52·19	32	− ·900	+ ·386	+ ·060	290 22	−1 9	+0·07	+0·4
302	Bamberg	11 50 40·77	30	− ·892	+ ·406	+ ·086	291 33	−1 10	+0·38	+0·3
303	Munich	12 13 48·55	36	− ·892	+ ·408	+ ·008	291 38	−1 5	+0·55	+0·2
304	Bamberg	12 34 25·15	45	− ·887	+ ·419	− ·010	292 12	−1 3	+1·77	+0·2
305	Dublin	12 4 34·8	45	− ·883	+ ·429	+ ·259	292 49	−1 18	+1·19	+0·1
306	Flower Observatory	12 25 39·37	80	− ·879	+ ·439	+ ·859	293 4	−1 50	+1·32	+0·2

307	Heidelberg, Astrometr. Abth.	12 7 36·82	36	− ·881	+ ·436	+ ·076	293 15	−1 8	+1·86	0·0
308	Dublin	12 14 3·7	46	− ·878	+ ·441	+ ·241	293 29	−1 16	+1·47	−0·1
309	Odessa	12 28 7·8	32	− ·875	+ ·449	− ·191	294 6	−0 52	+0·83	−0·1
310	Jena University	12 33 39·3	45	− ·861	+ ·480	+ ·028	295 51	−1 2	+0·99	+0·2
311	Bamberg	12 10 9·19	36	− ·857	+ ·487	+ ·079	296 23	−1 5	+1·08	+0·3
312	Nicolaieff	12 28 24·20	32	− ·855	+ ·493	− ·164	296 47	−0 50	+0·93	+0·4
313	Leipzig	11 51 2·07	30	− ·852	+ ·498	+ ·127	297 6	−1 7	+1·22	+0·4
314	Lyons	12 9 59·9	39	− ·849	+ ·504	+ ·108	297 22	−1 7	+0·76	+0·4
315	Turin	12 15 56·99	39	− ·848	+ ·508	+ ·058	297 36	−1 3	+0·65	+0·4
316	Leipzig	12 34 0·13	45	− ·846	+ ·510	+ ·043	297 44	−1 1	+1·10	+0·4
317	Durban	11 47 27·5	25	− ·830	+ ·541	− ·743	299 53	−0 18	+0·11	+0·5
318	Geneva	12 11 29·0	39	− ·826	+ ·547	+ ·112	300 6	−1 3	+0·16	+0·5
319	Nicolaieff	12 23 22·9	30	− ·798	+ ·595	− ·088	303 15	−0 49	+0·45	+0·6
320	Besançon	12 9 26·1	39	·795	+ ·600	+ ·153	303 29	−1 2	+0·79	+0·6
321	Toulouse	11 40 37·82	34	·793	+ ·603	+ ·235	303 47	−1 7	+0·60	+0·6
322	Zürich	12 13 7·7	39	·771	+ ·637	+ ·140	305 58	−0 58	+0·62	+0·6
323	Washington	12 5 41·5	75	− ·762	+ ·648	+ ·910	306 28	−1 39	+1·56	+0·5
324	Königsberg	12 0 39·9	32	− ·765	+ ·644	+ ·166	306 34	−0 59	+0·93	+0·6
325	Warsaw	12 0 38·8	30	− ·765	+ ·645	+ ·130	306 36	−0 57	+1·71	+0·6
326	Flower Observatory	12 7 16·79	75	− ·749	+ ·667	+ ·895	307 44	−1 37	+1·14	+0·5
327	Princeton	12 7 53·3	75	− ·743	+ ·675	+ ·890	308 19	−1 36	+1·80	+0·6
328	Strassburg	12 9 33·25	39	− ·734	+ ·686	+ ·193	309 26	−0 57	+1·13	+0·6

Table of Computed Values of the Vertical Distance v of the Lunar Surface above or below the Surface of a Sphere of Radius 15′ 32″·50—continued.

No.	Place.	G.M.T.	Star.	*a*.	*b*.	*c*.	P.	D.	*v*.	*v*′.
		h m s		″	″	″	° ′	° ′	″	″
329	San Fernando	11 39 3·4	41	−0·694	+0 733	+0·278	312 54	−0 58	+0·99	+0·6
330	Heidelberg, Astrophys. Abth.	12 9 4·56	39	− ·692	+ ·736	+ ·223	313 6	−0 54	+0 81	+0·6
331	Turin	11 48 55·08	34	− ·692	+ ·737	+ ·216	313 11	−0 54	+1·20	+0·6
332	Lyons	11 42 40·9	34	− ·687	+ ·742	+ ·264	313 35	−0 56	+0·69	+0·7
333	Königsberg	11 54 33·1	30	− ·683	+ ·746	+ ·244	313 58	−0 54	+1·22	+0·7
334	Flower Observatory	12 39 48·64	89	− ·676	+ ·755	+ ·828	314 5	−1 26	+1·00	+0·7
335	Königsberg	12 35 8·2	45	− ·668	+ ·762	+ ·204	315 5	−0 51	+1·38	+1·0
336	Geneva	11 43 46·9	34	− ·652	+ ·779	+ ·270	316 29	−0 52	+1·16	+1·0
337	Bamberg	12 10 42·89	39	− ·648	+ ·782	+ ·237	316 43	−0 50	+1·04	+1·0
338	Königsberg	12 11 4·6	36	− ·607	+ ·821	+ ·277	320 0	−0 48	+0·36	+0·7
339	Dublin	11 39 19·4	39	− ·597	+ ·829	+ ·472	320 40	−0 57	+0·30	+0·4
340	Besançon	11 40 48·8	34	− ·596	+ ·830	+ ·310	320 46	−0 48	+0·05	+0·4
341	Jena University	12 8 43·2	39	− ·589	+ ·836	+ ·282	321 18	−0 46	−0·32	+0·1
342	Jena (W. Winkler)	12 8 40·1	39	− ·588	+ ·837	+ ·283	321 23	−0 46	−0·82	+0·1
343	Zürich	11 44 12·7	34	− ·563	+ ·857	+ ·300	323 16	−0 44	+0·00	−0·6
344	Leipzig	12 8 28·04	39	− ·557	+ ·862	+ ·301	323 39	−0 43	+0·03	−0·7
345	Christiania	12 14 5·6	45	− ·557	+ ·862	+ ·424	323 39	−0 51	+0·12	−0·7
346	,,	12 22 17·3	46	− ·536	+ ·877	+ ·427	325 5	−0 49	−0·10	−1·1
347	Strassburg	11 39 26·4	34	− ·495	+ ·906	+ ·350	328 6	−0 39	−0·42	−1·3

348	Berlin	12 5 48·3	39	− ·473	+ ·919	+ ·355	329 31	−0 38	−0·87	−1·4
349	Odessa	12 37 18·5	39	− ·465	+ ·924	+ ·186	330 5	−0 28	−0·86	−1·3
350	Toulouse	11 13 33·1	31	− ·465	+ ·924	+ ·367	330 9	−0 37	−2·43	−1·4
351	Madrid	11 46 50·8	43	− ·458	+ ·928	+ ·303	330 29	−0 34	−2·21	−1·2
352	Pulkovo	11 54 56·8	32	− ·436	+ ·941	+ ·442	332 5	−0 38	−1·22	−0·9
353	Heidelberg, Astrometr. Abth.	11 37 49·61	34	− ·422	+ ·948	+ ·377	332 58	−0 34	−0·87	−0·7
354	Strassburg	12 39 27·3	53	− ·374	+ ·972	+ ·327	335 56	−0 27	+0·32	−0·2
355	Bamberg	11 38 28·24	34	− ·351	+ ·982	+ ·394	337 36	−0 26	+0·40	−0·3
356	Warsaw	12 12 22·9	39	− ·348	+ ·984	+ ·380	337 44	−0 27	+0·40	−0·3
357	Heidelberg, Astrometr. Abth.	12 36 37·05	53	− ·285	+1·01	+ ·372	341 35	−0 21	+0·70	−0·4
358	Leipzig	11 32 25·44	34	− ·163	+1·04	+ ·451	349 10	−0 11	+0·58	+0·3
359	Dublin	12 4 8·0	53	− ·144	+1·04	+ ·503	350 8	−0 13	+0·20	+0·3
360	Königsberg	12 0 20·7	39	− ·128	+1·05	+ ·502	351 12	−0 11	+0·04	+0·3
361	Liverpool	12 6 29·1	53	− ·078	+1·05	+ ·499	354 6	−0 6	+0·12	+0·2
362	Madrid	12 13 2·9	58	− ·060	+1·05	+ ·289	355 8	+0 6	+0·54	0·0
363	Nicolaieff	11 58 35·30	34	+ ·054	+1·05	+ ·457	362 3	+0 9	+0·24	−0·4

Notes.

Nos. 15 and 170, ten seconds added to recorded time.
No. 96, thirty seconds subtracted from recorded time.
Nos. 185 and 190, one minute subtracted from recorded time.
Nos. 280 and 298, one and two seconds respectively subtracted as suggested by the observer's notes.

Mean Values of $v-v'$.

P.			$v-v'$.	P.			$v-v'$.
°		°	"	°		°	"
...			...	362	to	340	+ 0·30
15	to	30	+ 0·26	340	,,	330	− 0·02
30	,,	40	+ 0·13	330	,,	320	+ 0·22
40	,,	50	− 0·15	320	,,	310	+ 0·29
50	,,	60	− 0·26	310	,,	300	+ 0·42
60	,,	70	− 0·26	300	,,	290	+ 0·64
70	,,	80	− 0·02	290	,,	280	− 0·06
80	,,	90	+ 0·08	280	,,	270	+ 0·02
90	,,	100	+ 0·04	270	,,	260	− 0·06
100	,,	110	+ 0·62	260	,,	250	+ 0·48
110	,,	120	+ 0·01	250	,,	240	+ 0·42
120	,,	130	0·00	240	,,	230	− 0·10
130	,,	140	+ 0·37	230	,,	220	+ 0·43
140	,,	150	− 0·05	220	,,	210	− 0·29
150	,,	160	+ 0·72	210	,,	200	− 1·08
160	,,	180	+ 0·78	200	,,	180	− 0·64

Uplands, Cobham, Surrey:
1911 *May* 5.

Requests to Observers of Comets. By Professor K. Pokrowski.

[Abstract of a note in *A.N.*, 4485, translated at the request of the author, and communicated by the Secretaries.]

In researches on the forms of comets and the movements of cometary matter, it is often impossible to make use of otherwise excellent observations in consequence of the insufficiency of the data given.

Thus, the approximate velocity of motion of cometary masses from the nucleus is recorded, but not the position angle; many drawings have no scale; the length and breadth of the tail may be given, but not its exact direction; stars are mentioned to give the position of the tail, but it is not stated whether they are in its central line or on the preceding or following edge.

Observers of comets are especially requested to attend to the following points:—

1. In micrometrical observations of the head, the length and position angle of the streamers from the nucleus should be given, as well as the coordinates of some points in the circumference in relation to the nucleus, or at least the distance of the nucleus from the vertex of the head.

In drawings, the scale should not be forgotten.

2. In observations of anomalous tails (directed towards the Sun), the length and curvature of the tail should be determined by coordinates of some points.

3. In the case of condensations in the substance of the tail, the coordinates of each should be given for as many instants as possible.

4. In visual observations it is better to trace the outline on a star map. Where the drawings cannot be published, the coordinates of some points along the central line of the tail should be given. In the case of divided tails, the central line of each streamer should be similarly determined.

[The above remarks are based upon the author's experience in discussing observations of Comet 1910 *a*. See his paper, *Der Schweif des Kometen* 1910 *a*; Jurjew (Dorpat), 1911.]

Positions of Halley's Comet and of Comet 1910 *a from photographs taken at the Khedivial Observatory, Helwân.*
By H. Knox Shaw, B.A.

The photographs were taken with the Reynolds 30-inch reflector; and as these are the first results of this nature to be published from this observatory, a short description of the methods of measurement and reduction employed will be given.

The focal length of the reflector is 3495 mm., giving a scale of 1 mm. = 59″ on the plate. The plates, after having a *réseau* impressed upon them or bound to them, have been measured in four orientations in a measuring machine made by the Cambridge Scientific Instrument Company, of a form similar to, but less complex than, that described by Mr. Hinks in *M.N.*, lxi. p. 444. The unit to which the measurement is carried is $\frac{1}{2000}$ mm., equivalent to 0″·03.

Continuous exposures were given to all these plates; but when the comet is moving rapidly and the star trails are necessarily long, the method of breaking the exposure in the middle described by Dr. Rambaut in *M.N.*, lxx. p. 497, is preferable, especially when at either the beginning or the end of the exposure the comet is at a low altitude, as then the ends of the star trails are irregular and difficult to set upon in the measuring machine. This method has been since adopted, except that measurement is made to the middle of the break, and not to the extremities of the images at each side of it. Of the plates considered here, only on that of October 22, and possibly on that of October 15, when the exposures were unnecessarily prolonged, are the star trails inconveniently long. The guiding was done, with

the exception of the photograph of Comet 1910 *a*, when recourse was had to the guiding telescope and moving wire, by an eye-piece at the side of the plate, which can be moved relatively to the plate by micrometer screws both in R.A. and declination. The plate was made to follow the comet in each case.

The places of the comparison stars have been taken from the catalogues of the Astronomische Gesellschaft, and Boss's systematic corrections to the A.G. zones have been applied. All the available stars near enough to the comet have been employed, but in one or two cases it was considered better to reject one of the stars and make a fresh reduction without it, as a large residual seemed to indicate a proper motion. Such, for instance, were A.G. Leipzig II. 444 and Berlin A. 2042, for which centennial P.Ms. of about 16″ and 11″ respectively were deduced.

The method of reduction employed is as follows:—The second order terms in the differential refraction are, when they are sensible, first applied to the measured coordinates, and then these are connected to the standard coordinates of the comparison stars by equations of the form

$$\xi = x + ax + by + c$$
$$\eta = y + dx + ey + f$$

—these equations being solved by least squares to give the six plate constants a, b, c, d, e, f.

It does not seem to be sound to publish positions of a comet to a tenth of a second of arc without attempting to give some indication, even if it be only a personal opinion, of the value of the last figure. The error in the deduced position is due both to error in its measured coordinates and to error in the plate reduction. The probable error of the measured coordinates of the comet has been found from consideration of all the settings on the comet, usually twelve in each coordinate, the difference between the measurements in the direct and reversed positions of the plate being considered as accidental, which it is probably not entirely. The value of this probable error is rarely greater than ±0″·1, but the measured coordinate is probably affected by a systematic error—due to asymmetry of the comet's image—which is larger than this. The probable error of the standard coordinates of the comet dependent upon the plate reduction has been deduced from a paper by Mr. Filon in *M.N.*, lxii. p. 561, where he gives it as

$$\pm 0{\cdot}6745\sqrt{\frac{[v^2]}{n(n-3)}}\sqrt{\left(2\frac{k'^2}{k^2}+1\right)},$$

where n is the number of comparison stars, $[v^2]$ the sum of the squares of their residuals in the plate reduction, and k, k' represent the scatter on the plate of the comparison stars, and of the stars

whose positions are to be determined. This has its minimum value when the comet lies at the centre of gravity of the comparison stars, k' becoming zero. With only a few comparison stars available, it is not always possible to satisfy this condition even approximately, and so the value $k' = \frac{1}{2}k$ has been adopted, and the value of the probable error becomes

$$\pm 0\cdot 83\sqrt{\frac{[v^2]}{n(n-3)}}.$$

This is affected both by the errors in the places of the comparison stars brought up to 1910·0 and by the errors made in their measurement. The probable error of a deduced coordinate due to both these causes is thus

$$\pm\sqrt{\left\{(0\cdot 67)^2\frac{[u^2]}{m(m-1)}+(0\cdot 83)^2\frac{[v^2]}{n(n-3)}\right\}},$$

m being the number of settings on the comet in that coordinate and $[u^2]$ the sum of the squares of their residuals from the mean. The actual error of the comet's place exceeds the probable error thus calculated, mainly owing to differential systematic error between the measurement of the comet's image and the star trails, caused by bad guiding or irregularity in the star trails when the exposure has been either begun or finished at a low altitude. It seems impossible to judge of the amount of this error. And thus it may be argued that the probable errors given in this paper are of little value. At any rate they show that the last figures given in the comet's place are not reliable, but are worth inclusion.

The positions given are referred to the equator and equinox of 1910·0. They are corrected for the aberration arising from the Earth's motion, but not for the aberration arising from the Comet's motion, nor for its parallax. Thus to obtain the comet's mean place for 1910·0 it is necessary—

(*a*) To apply the correction for parallax.

(*b*) To re-apply the correction for Earth's aberration, with the sign as given in the last column.

(*c*) To antedate for the light-time.

[Table.

Halley's Comet.

Date and G.M.T.	R.A. 1910·0.	Log $p.\Delta.$	Dec. 1910·0.	Log $p.\Delta.$	Corr. for Earth's Aberration. R.A.	Corr. for Earth's Aberration. Dec.	Pl.
1909. h m s	h m s		° ′ ″		s	″	
Aug. 24 13 6 0	6 13 1·98±0·015	9·707n	+17 16 26·0±0·22	0·542	−0·75	+2·1	(*a*)
Sept. 13 12 40 0	6 18 36·08±0·006	9·669n	+17 10 35·2±0·15	0·486	−0·34	+2·3	(*b*)
15 13 2 0	6 18 51·02±0·007	9·632n	+17 9 48·7±0·21	0·451	−0·29	+2·3	(*c*)
16 12 37 0	6 18 56·73±0·009	9·659n	+17 9 26·1±0·16	0·476	−0·27	+2·3	(*d*)
Oct. 15 12 5 4	6 11 10·02±0·014	9·499n	+16 58 28·8±0·16	0·374	+0·47	+2·0	(*e*)
22 11 27 42	6 4 53·41±0·024	9·508n	+16 56 19·0±0·29	0·380	+0·67	+2·0	(*f*)
1910.							
Jan. 27 5 41 8	1 10 51·90±0·010	9·561	+ 8 37 18·4±0·20	0·544	−0·39	−2·0	(*g*)
27 5 43 8	1 10 51·73±0·006	9·565	+ 8 37 18·0±0·29	0·545	−0·39	−2·0	,,
27 5 45 8	1 10 51·67±0·010	9·569	+ 8 37 17·5±0·18	0·546	−0·39	−2·0	,,
27 5 47 8	1 10 51·49±0·012	9·572	+ 8 37 17·7±0·18	0·546	−0·39	−2·0	,,

Comet 1910 a.

1910.							
Apr. 11	14 6 52	22 39 18·57±0·006	9·727n	+21 22 40·1±0·23	0·538	−0·99	−11·8	(*h*)

The following were used as comparison stars in reducing the plates:—

Plate (*b*) A.G. Berlin A. 2099, 2111, 2115, 2117, 2132.

(*c*) ,, ,, ,, 2111, 2115, 2117, 2132.

(*d*) ,, ,, ,, 2099, 2111, 2115, 2117, 2132.

(*e*) ,, ,, ,, 2016, 2018, 2019, 2021, 2038, 2051, 2052, 2054.

(*f*) ,, ,, ,, 1939, 1940, 1941, 1943, 1949, 1953, 1960, 1965.

(*g*) A.G. Leipzig II. 431, 439, 440, 441, 447, 460.

(*h*) A.G. Berlin B. 8719, 8720, 8732, 8742.

Notes on the Plates.

(*a*) This plate was first measured at the Royal Observatory, Greenwich, and the result published in *A.N.*, 4386, where the position was given uncorrected for the aberration arising from the Earth's motion. It was not possible to repeat the measurement at Helwân in the same way owing to the plate having been broken, but the plate was reduced by means of eight comparison stars of about the twelfth magnitude distributed about the comet. The

positions of these eight stars were deduced from another plate of the same field taken with the reflector, this plate being reduced by means of the four stars, A.G. Berlin, A. 2038, 2051, 2054, 2061. The positions deduced at Greenwich and Helwân agree—when allowance is made for Boss's systematic corrections—within $0^s.05$ and $0''.2$, which is very fair agreement, considering the extreme faintness of the comet's image. The magnitude of the comet deduced from this plate * is $15\frac{1}{2}$–16.

(*b*), (*c*), and (*d*). These are all good plates, the comet's image being quite strong on each of them. Magnitude of comet 15.

(*e*) and (*f*). Unnecessarily long exposures were given to these plates, and the star trails are difficult to measure, especially on the latter plate. This fact probably causes the large probable error, which is entirely due to that arising from the bad fit given by the plate constants. Magnitude of comet, $14\frac{1}{2}$ on October 15 and 14 on October 22.

(*g*) Four exposures on the same plate. The plate constants were determined for each exposure separately. The values of these plate constants may be of interest as giving an indication of the accuracy attainable in the measurement of such photographs.

	Exp. 1.	2.	3.	4.
a	−0·01576	1575	1565	1581
b	+0·01113	1105	1095	1091
d	−0·01041	1030	1063	1057
e	−0·01491	1504	1504	1514

The above is a very favourable instance, as the star images were circular—the exposures were only two minutes and thus easy to measure.

(*h*) Three plates were obtained of comet 1910 *a* in April, but only this one was considered good enough to measure. Magnitude of comet, $13\frac{1}{2}$.

* *M.N.*, lxxi. p. 232.

Khedivial Observatory, Helwân:
1911 *April* 22.

Rotation of Stars about their Axes. By George Forbes, F.R.S.

No one has ever published observations to define the axis and speed of rotation of a star. The purpose of the present note is to draw attention to special cases where this can be done by the same spectroscopic method as has been used with so much success on the solar rotation.

If the axis of rotation of a star be at right angles to the line of sight, approximately, the lines of its spectrum must be widened at both sides by the Doppler effect, as indicated by Abney in 1877 (*M.N.*, xxxvii. p. 278).

In the case of an Algol variable, if, as in the solar system, the axis be approximately at right angles to the place of the orbit, then at the early part of the eclipse the more, or the less, refrangible part of the widened lines must be eclipsed according as the rotation is in the same direction as the revolution or in the opposite direction.

The other side of the widened line is eclipsed at the later part of the eclipse. Thus the spectrum lines seem to be displaced.

It is known that the positions of the spectrum lines of Algol are affected by the revolution. But, if attention be directed to it, there is a great probability of the same kind of effect being visible during the progress of the light-minimum. If in Algol or any other variable of the Algol type the effect be observed as described above, it will be possible to fix with a fair degree of accuracy the direction of the axis of rotation of the luminous partner and the velocity of the equatorial parts.

The writer has not the means of testing this, and publishes this note in the hopes that it may lead to the discovery by others who are better equipped.

Pitlochry:
1911 *April* 26.

Parallax of 17 Lyræ C. By Frederick Slocum.

(*Communicated by the Secretaries.*)

In the *Monthly Notices* of R.A.S., lxviii. p. 517, and lxxi. p. 208, Professor Burnham has called attention to a faint star with large proper motion near 17 Lyræ (R.A. $19^h\ 4^m$, Dec. $+32°\ 21'$). In 1907 this star was put upon the parallax observing list for the 40-inch refractor, and nine plates were obtained at six parallactic epochs, as shown in Table I.

Table I.

Plate Number.	Date.	Hour Angle. h	Observer.	Quality.
10 A	1907 June 11	−2·0	F	Good
77	1908 Nov. 28	+3·6	F, A	Good
90	1909 Mar. 20	−1·7	F	Good
105	Sept. 11	0·0	Sl	Fair
245	1910 Apr. 9	−1·4	Su	Good
252	May 14	−0·3	Su	Good
280	Aug. 6	−0·2	Su	Good
282	Aug. 20	−0·4	Sl, Su	Good
288	Aug. 26	−0·4	Su, Su	Fair

The Hour Angle column indicates the position of the telescope at the middle of the exposure. A plus sign means that the telescope was pointing west of the meridian. Among the observers F = Fox, A = Abetti, Sl = Slocum, Su = Sullivan. Two exposures were made on plates 77, 282, and 288; one on each of the others. The average exposure time was fifteen minutes.

In the present series a Wallace visual colour-filter and Cramer Isochromatic plates were used. In other respects the process of making the plates is as described by Professor Schlesinger in the *Astrophysical Journal*, xxxii. 372, 1910.

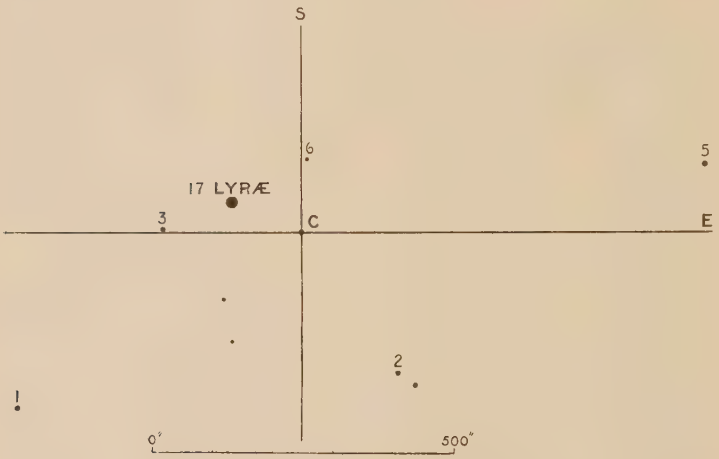
Five comparison stars were used, the magnitudes and relative positions of which are given in Table II. The approximate standard coordinates are taken from the measures of plate 282.

Table II.

Plate Number.	Diameter.	Visual Magnitude.	Photographic Magnitude.	X.	Y.
	mm.			″	″
1	0·23	10·5	11·2	−501·89	+239·57
2	·27	10·0	10·0	+145·28	+178·87
3	·36	9·3	9·2	−251·85	− 61·62
5	·32	9·6	9·6	+611·24	−173·54
6	·17	11·4	11·5	− 2·88	−183·29
Parallax Star	·17	11·3	12·5	− 14·93	− 59·16

The accompanying diagram shows the relative positions of 17 Lyræ, the parallax star C, and the comparison stars.

In Professor Burnham's first paper the magnitude is given as 12·00, from a photographic determination by Mr. Jordan. In the second paper it is referred to as an "eleventh magnitude" star, this being Professor Burnham's visual estimate. The discrepancy is due to the fact that the star is red, the difference between the visual and photographic magnitudes being 1·2, as shown in the table. The magnitudes given in Table II. were obtained from photographs which I have taken for the purpose with the 24-inch



reflector: the photographic magnitudes by using ordinary plates sensitive to the blue rays; the visual magnitudes by using a visual colour-filter and plates sensitive to the red end of the spectrum. Both sets of measures were based upon the following photometric standards on the international system, kindly furnished me by Mr. Parkhurst.

B.D. Number.	Magnitude.	
	Visual.	Photographic.
32° 3321	8·13	9·29
32° 3323	8·32	8·49
32° 3325	9·26	...
32° 3326	...	5·40

Table III. gives the values used in the reduction.

TABLE III.

Plate Number.	Solution.	Correction for μ.	Corrected Solution.	Weight p.	Parallax Factor.	Time in Years	ν $p.v.$
	$''$	$''$	$''$				$''$
10 A	−2·8222	+2·8565	+0·0343	1	+0·418	−2·3111	−0·031
77	−1·1497	+1·0448	− ·1049	1·5	− ·607	− ·8453	+ ·023
90	·5391	+ ·0657	+ ·1266	1	+ ·959	·5386	− ·014
105	− ·2129	+ ·0732	− ·1397	0·9	− ·898	− ·0592	+ ·024
245	+ ·7739	− ·6404	+ ·1335	1	+ ·992	+ ·5181	+ ·007
252	+ ·8376	− ·7585	+ ·0791	1	+ ·782	+ ·6137	+ ·038
280	+1·0229	−1·0423	− ·0194	1	− ·490	+ ·8433	− ·020
282	+1·0274	−1·0895	− ·0621	1·5	− ·679	+ ·8815	− ·001
288	+1·0583	−1·1057	− ·0514	1·3	− ·749	+ ·8978	− ·024

Each plate furnishes a residual or "solution" for the parallax star. This residual is corrected for proper motion by using Professor Burnham's value $1''·658$ in $48°·2$, or rather the component of that proper motion which is parallel to the equator. Weights were assigned according to the quality of the plate and the number of exposures. Each plate yields an equation of the form

$$C + T d\mu + P\pi = m$$

in which C is the correction to the standard X-coordinate, T the time in years, $d\mu$ the correction to the proper motion, P the parallax factor, π the parallax, and m the solution of the plate. The nine equations thus formed and weighted give the following normals:—

$$\begin{aligned} 9·90\ C - 0·2169\ d\mu - 0·8242\ \pi &= -0·0660 \\ -0·2169\ C + 11·2508\ d\mu + 2·0312\ \pi &= -0·0014 \\ -0·8242\ C + 2·0312\ d\mu + 5·4562\ \pi &= +0·6436 \end{aligned}$$

From the least square solution

$$\begin{aligned} C &= +0''·003 \\ d\mu &= -0''·023 \\ \pi &= +0''·127 \pm 0''·008 \end{aligned}$$

The residuals for the comparison stars give no indication of either proper motion or parallax. Some idea of the degree of accuracy with which plates made with this telescope (focal length 19 metres) can be measured may be obtained from Table IV., which gives the mean of the residuals of the comparison stars for each plate, taken without regard to sign.

TABLE IV.

10A	77	90	105	245	252	280	282	288	Mean of all
$0''·027$	·008	·017	·006	·011	·017	·024	·027	·016	·017

The Yerkes Observatory:
1911 *April* 18.

Fourth Note on the Number of Faint Stars with Large Proper Motions in the Oxford Zones. By F. A. Bellamy, M.A.

Previous papers upon this subject have been communicated to the Society by Professor Turner in November 1908 (*M.N.*, lxix. 57), April 1909 (*M.N.*, lxix. 491), and November 1910 (*M.N.*, lxxi. 45). The present note, the fourth of the series, contains the results of probable P.Ms. for twenty-four plates compared in zone + 29°. The interval between the early and late plates is little more than the ten years' limit adopted for the comparison of plates in these investigations; all are between 10·19 and 12·03 years.

The Government Grant Fund afforded the necessary money for the preliminary calculations required in the comparison of the two sets of twenty-four plates. All the unidentified stars, the apparent discordances, and differences amounting to ·005 (1″·5) between the two plates, in either x or y coordinate, 700 in number, have been examined, and the stars remeasured by myself, usually on both plates and in the direct and reversed positions of the plate. With a few exceptions, all stars with a final difference of not less than ·005 in either x or y, ·004 and ·004, or some with ·004 and ·003, have been included in this paper: the exceptions are some stars with very faint or elongated images near the margins of the plates.

The number of pairs of stars compared on the twenty-four plates is 3571, with a mean interval of 10·69 years, and the stars with probable large P.Ms. amount to 80. This number is very similar to the relative number of stars compared and P.Ms. which resulted in zones + 26°, + 27°, and + 28°; a summary of the P.Ms. for + 29° is given in Table I., and for comparison the totals for the other zones examined are given at the end of the Table.

These results were then collected and the average number of P.Ms. per plate (approximately $4\frac{1}{3}$ square degrees in area) was obtained, as in the first portion of Table II.; the results are in fair agreement with similar results for zones + 26°, + 27°, and + 28°, allowance being made for the paucity of material. From the comparison of 93 plates, not unevenly distributed in all hours of R.A., and with an interval between the early and late plates varying between 10 and 15 years, rather more than three P.Ms., exceeding 12″·5 per century, may be expected per plate of $4\frac{1}{3}$ sq. deg.; 1·14 P.Ms. for those stars with P.Ms. between 15″ and 20″ per century; and 0·86 for those stars with P.Ms. exceeding 20″.

TABLE I.

Zone +29°. Stars with probable Large P.Ms.

R.A. of Plate Centre.	Total Stars compared.	Interval in Years.	Total P.Ms.	Centennial P.Ms. 15″ to 20″.	Centennial P.Ms. >20″.
h m					
7 7	195	10·19	11	6	3
7 16	138	10·19	6	4	2
7 25	145	10·19	5	1	4
9 4	105	10·13	0	0	0
9 49	83	10·14	2	0	1
9 58	60	10·13	3	2	1
10 7	58	10·13	2	1	1
10 16	57	10·13	2	1	1
10 34	67	10·13	4	4	0
14 28	114	10·14	0	0	0
14 37	85	10·14	3	0	3
14 46	102	10·14	2	1	0
14 55	125	10·14	0	0	0
16 25	83	11·19	0	0	0
17 19	145	11·01	5	2	2
17 28	204	11·01	1	0	1
17 46	184	11·00	0	0	0
17 55	203	10·99	3	0	0
18 4	265	10·98	0	0	0
18 49	301	11·17	6	2	1
19 52	336	11·29	6	2	3
22 43	172	12·03	8	6	2
23 19	128	12·03	9	2	2
23 55	179	12·03	2	2	0
+29° 24 plates	3,534	...	80	36	27
+26° 30	6,510	...	101	32	20
+27° 29	3,751	...	84	28	27
+28° 10	2,822	...	50	10	6
Totals 93	16,617	...	315	106	80

The second and third portions of Table II. are intended to show the general distribution of these P.Ms. depending upon the galactic latitude: though the P.Ms. are widely distributed over all galactic latitudes, it is sufficient for this purpose if only two groups be made, within and without the Milky Way, as in the following Tables II. and III.

TABLE II.

Total.				Within the Milky Way.				Without the Milky Way.			
Plates.	P.Ms.	Centennial P.Ms. 15″–20″.	>20″.	Stars.	All P.Ms.	15″–20″.	>20″.	Stars.	All P.Ms.	15″–20″.	>20″.
+26° 30	3·37	1·07	0·67	3842	41	15	7	2668	60	17	13
+27° 29	2·90	0·96	0·90	937	13	4	2	2814	71	24	25
+28° 10	3·50	1·00	0·60	1585	21	5	2	1237	14	5	4
+29° 24	3·33	1·50	1·13	1115	34	15	13	2419	46	21	14
93	3·39	1·14	0·86	7479	109	39	24	9138	191	67	56
Sum.	Weighted Means.				Sums.				Sums.		

It appears from Table II. that, with approximately the same number of stars, we may expect nearly twice as many large P.Ms. out of the Milky Way as within its limits. If we put this information in the form of ratios, P.Ms. to stars compared, the results are shown with their proper values, as may be seen in Table III.; the 29° zone gives results in the opposite direction, which may be due partly to the accidental distribution of the plates compared and partly to the numbers used, which are comparatively small.

TABLE III.

Ratios of P.Ms. to Stars.

	Within the Milky Way.			Without the Milky Way.		
	All P.Ms.	15″–20″.	>20″.	All P.Ms	15″–20″.	>20″.
+26°	1 : 94	1 : 256	1 : 549	1 : 45	1 : 157	1 : 205
+27°	72	234	469	40	117	113
+28°	75	317	793	88	247	309
+29°	34	74	86	53	115	173
26° to 29°	1 : 69	193	312	1 : 48	136	163

The results of the comparison of the 24 plates in zone +29° are given in detail in Table IV. in a form similar to those in previous notes.

TABLE IV.

List of Large P.Ms. in Zone +29°.

R.A. of Plate Centre.	Oxford. +29° No.	Mag. (Photo.)	Cent. P.M. in Arc.	Δx.	Δy.	Interval.
h m			″			y
23 19	68247*	8·4	75·4	+·030	−·004	12·03
23 19	68304	11·6	58·8	+ 19	− 14	12·03
22 43	66606*	6·0	44·5	− 11	− 14	12·03
9 49	28124*	8·2	38·5	− 12	− 5	10·14
17 28	41611*	9·5	32·9	− 5	− 11	11·01
19 52	55709*	6·8	31·9	+ 1	+ 12	11·29

TABLE IV.—*continued.*

R.A. of Plate Centre.	Oxford. +29° No.	Mag. (Photo.)	Cent. P.M. in Arc.	Δx.	Δy.	Interval.
h m			″			y
9 58	28516*	7·9	30·2	− 10	− 2	10·13
7 7	20806	11·9	29·4	+ 7	+ 8	10·19
17 19	41073	10·8	27·2	0	− 10	11·01
7 25	21697	11·1	26·6	− 9	0	10·19
7 7	20453	11·9	25·8	− 8	+ 4	10·19
10 16	29384*	10·7	25·5	− 5	− 7	10·13
10 7	29019*	10·7	24·3	− 2	− 8	10·13
7 25	21472	10·2	24·2	+ 8	+ 2	10·19
7 25	21533	11·9	24·2	− 8	+ 2	10·19
19 52	55774*	7·4	24·2	+ 2	− 9	11·29
14 37	35936	11·1	24·0	− 8	− 1	10·14
7 16	21220	11·4	23·9	− 8	− 1	10·19
18 49	46319	11·5	23·1	+ 5	− 7	11·17
22 43	66516*	7·4	22·5	+ 9	0	12·03
14 37	35960*	7·2	21·6	− 7	+ 2	10·14
7 16	20931*	4·1	21·5	− 7	− 2	10·19
19 52	55480*	6·5	21·5	− 4	− 7	11·29
14 37	36036*	5·8	21·3	− 4	− 6	10·14
17 19	41252	12·4	21·2	+ 5	+ 6	11·01
7 7	20550	9·7	20·9	− 1	− 7	10·19
7 25	21644*	10·0	20·9	− 7	+ 1	10·19
23 55	70145	10·8	19·9	0	+ 8	12·03
9 58	28804	11·6	19·8	− 6	− 3	10·13
10 7	29020	11·2	19·8	− 3	− 6	10·13
10 34	29966*	6·9	19·8	+ 6	− 3	10·13
10 34	30065	12·0	19·8	− 6	+ 3	10·13
19 52	55771*	8·0	19·4	+ 2	− 7	11·29
7 16	21260	11·4	18·9	+ 4	− 5	10·19
7 16	21346	11·4	18·9	− 4	− 5	10·19
19 52	55355	10·4	18·6	− 7	0	11·29
7 16	20977	11·4	18·6	+ 2	+ 6	10·19
7 7	20606	11·3	18·5	+ 2	− 6	10·19
7 7	20655	12·1	18·5	− 2	− 6	10·19
22 43	66638	12·2	18·3	+ 2	+ 7	12·03
9 58	28533	12·0	18·1	− 1	− 6	10·13
14 46	36138*	7·3	18·1	6	− 1	10·14
18 49	46540	10·1	18·0	− 6	+ 3	11·17
22 43	66151*	8·0	18·0	− 4	− 6	12·03

TABLE IV.—*continued.*

R.A. of Plate Centre.	Oxford. +29° No.	Mag. (Photo.)	Cent. P.M. in Arc.	Δx.	Δy.	Interval.
h m			″			y
7 7	20734	11·3	17·9	− 1	+ 6	10·19
22 43	66642*	8·6	17·8	− 7	− 1	12·03
23 19	68202	12·0	17·7	− 5	− 5	12·03
23 19	68354	11·6	17·7	− 7	+ 1	12·03
23 55	70156*	8·5	17·7	+ 5	− 5	12·03
7 16	21243	10·6	17·1	− 5	+ 3	10·19
10 34	29934	11·6	16·9	+ 4	− 4	10·13
7 7	20426	11·5	16·8	+ 4	+ 4	10·19
7 7	20682	12·1	16·8	+ 4	+ 4	10·19
18 49	46654*	8·4	16·4	− 6	− 1	11·17
10 16	29458*	11·5	16·0	+ 2	+ 5	10·13
10 34	29937	11·6	16·0	− 5	− 2	10·13
22 43	66123*	9·6	16·0	− 4	− 5	12·03
7 25	21438	11·1	15·9	+ 2	− 5	10·19
17 19	41257	11·8	15·8	− 5	+ 3	11·01
22 43	66562*	9·6	15·7	− 2	− 6	12·03
17 19	41188	11·8	15·2	− 4	+ 4	11·01
22 43	66535*	10·4	15·2	+ 6	− 1	12·03
7 7	20615	12·1	15·0	− 1	− 5	10·19
23 19	68225*	5·7	14·9	+ 5	− 4	12·03
23 19	68274*	8·0	14·9	+ 6	0	12·03
14 46	36203	10·8	14·8	+ 4	− 3	10·14
9 49	28235	11·4	14·8	− 5	0	10·14
7 7	20526	12·4	14·7	− 5	0	10·19
7 7	20715*	10·2	14·7	0	+ 5	10·19
18 49	46509	10·3	14·5	+ 5	+ 2	11·17
18 49	46554	9·1	14·5	− 5	+ 2	11·17
17 19	41008	12·4	13·9	− 1	− 5	11·01
17 55	43039*	9·3	13·9	− 5	− 1	10·99
17 55	43298*	9·0	13·9	+ 1	− 5	10·99
17 55	43293*	6·9	13·7	− 3	+ 4	10·99
18 49	46527	9·9	13·7	+ 5	− 1	11·17
23 19	68242*	8·7	13·4	− 2	− 5	12·03
19 52	55736*	6·6	13·3	0	− 5	11·29
23 19	68245	11·2	12·7	+ 5	− 1	12·03
23 19	68350	10·5	12·7	+ 1	+ 5	12·03

The first column is the plate centre in zone + 29° ; the second

column gives the number of the star printed in the Oxford Astrographic Catalogue, the * denoting that it is also in one of the A.G. Catalogues; the third column shows the magnitude of the star obtained from the formula given at the head of each plate; the fourth column has the centennial P.M.; the fifth and sixth give the difference in *réseau* intervals (unit ·001) between the two plates in x and y for the interval given in the seventh column.

Taking these 80 P.Ms. in $+29^\circ$ zone and forming them into three-hour groups we get consistent results, which appear to indicate that these P.Ms. are in R.A. uniformly distributed in amount. Too much stress must not, however, be placed upon this conclusion; for no plates have been compared in this zone for R.A. 0^h to 6^h, and the other groups depend upon 2 to 5 plates with 5 to 22 P.M. stars. Table V shows the agreement. The mean is large for group 21^h to 0^h as it contains two exceptionally large P.Ms.; in fact, no other P.M. exceeding 50″ per century has been found from a comparison of more than 16,000 stars in zones $+26^\circ$ to $+29^\circ$.

TABLE V.

R.A.	Plates.	P.M. Stars.	P.M. per Century.
h m			″
6– 9	3	22	20·0
9–12	5	13	21·5
12–15	2	5	20·0
15–18	3	9	18·6
18–21	2	12	19·1
21– 0	3	19	23·4
Sum or Wt mean	18	80	20·7

If now we group these stars according to magnitude we get results as follows, N denoting the *total* number as bright as, or brighter than, the reading:—

TABLE VI.

Showing P.Ms. for Successive Magnitudes.

Mag.	To 7·9.	8·9.	9·9.	10·9.	12·0.
N over 20″	9	13	17	23	33
N 15″–20″	1	5	5	9	30
N over 15″	10	18	22	32	63
log N over 15″	1·00	1·26	1·34	1·50	1·80
Diff.		0·26	0·08	0·16	0·30
Zone $+27^\circ$		0·36	0·17	0·17	0·14
Zones 26° and 28°		0·36	0·22	0·16	0·19
Means		0·33	0·16	0·16	0·21

University Observatory, Oxford:
1911 *May* 12.

The Galactic Distribution of Spiral Nebulæ: Preliminary Note.
By Arthur R. Hinks, M.A.

1. In the discussion which followed the reading of a paper from the Royal Observatory, Greenwich, on Observations of Nebulæ, at the meeting of the Society 1911 April 12, I spoke, rather on the spur of the moment, as to the desirability of examining Mr. Franklin-Adams' plates for a classification of nebulæ. Since it seems probable that, with the consent of Mr. Franklin-Adams, this work will be undertaken, it may be useful to put on record the ideas which prompted the suggestion.

2. Let us divide the nebulæ into three classes:—

A. Extensive nebulæ, such as the Orion nebula, flocculent or curdled; the great nebula near ρ Ophiuchi, more uniform in substance, but with dark lanes; or the lacework nebulæ, such as Messier 1. These all lie in or near the Milky Way, and they are all presumably gaseous.

B. Small, well-defined, planetary or stellar nebulæ, which are all gaseous.

C. The so-called "white nebulæ," which give a continuous spectrum, and of which the large and better known examples are spirals.

It appears that by far the larger proportion of nebulæ belong to Class C, so that these have a predominating influence in any examination of distribution which does not discriminate between the different types.

3. Such examinations have been made—

(1) by Sir William Herschel, who is said by Proctor (*M.N.*, xxix. 339) to have "conceived that the nebulæ might be found to be arranged in a zone somewhat resembling the Milky Way, but nearly at right angles to it." [I have been unable to verify the reference.]

(2) by Sir John Herschel, who finds that "one-third of the whole nebulous contents of the heavens are congregated in a broad irregular patch occupying about one-eighth of the whole surface of the sphere; chiefly (indeed almost entirely) situated in the northern hemisphere . . . . In the southern a much greater uniformity of distribution prevails . . . . It must not be left out of consideration, and has been distinctly remarked by Sir William Herschel as an element of whatever speculation a closer attention to this subject and a more perfect classification of nebulous objects may lead us to indulge in, that the most condensed portion, and what may fairly be regarded as the principal nucleus, of the region of Virgo, is situated almost precisely in one pole of the Milky Way . . . . The phenomena on the other side of the Milky Way, though much less characteristic, are not altogether dissimilar." (Results, *C.G.H.*, pp. 134–137.)

(3) by Cleveland Abbe, who found "a very decidedly increasing condensation as we approach the poles of the Via Lactea." (*M.N.*, xxvii. p. 258.)

(4) by Proctor, who says: "It will be observed that there is no

mistaking the fact that a true zone of nebular dispersion (if I may use that term as the converse of aggregation) exists in the heavens. Nor can the exact agreement of this zone with the galactic zone be called in question." (*M.N.*, xxix. p. 340.)

(5) by Sidney Waters, who laid down all the objects in the N.G.C. on two well-known charts, (*M.N.*, liv. p. 526.) He says: "the remarkable avoidance of the nebulæ of the Galaxy . . . . is equally significant with the coincidence of the clusters with its main track."

(6) by Bauschinger, who from the N.G.C. concludes: "Die schwachen Nebel vermeiden die Milchstrasse; die grossten Anhäufungen derselben finden in der Nähe der Pole der Milchstrasse statt; von diesen Polen ausnimmt die Zahl der Nebel um so mehr ab, je näher man der Milchstrasse kommt. Ausserdem finden sich hiervon unabhängige Anhäufungen am südlichen Himmel in den Cap wolken, und am nördlichen ein Sternbild der Andromeda." (*V.J.S. Astr. Ges.*, **24**, p. 51.)

(7) by Stratonoff, who concludes: "En résumé, tout ce qui précède fait supposer que la distribution des nébuleuses est intimement liée a la voie lactée de manière qu'elles forment un système commun." (*Public. Observ. Tachkent*, No. 2, p. 49.)

(8) by Bohlin, who examined separately the round, the little extended, and the very extended nebulæ, and who found that "in every case the concentrations referred to are clearly visible, the dominions of the round nebulæ being the most vast, and the very extended nebulæ being the most constricted towards the poles of the 'Milky Way.'" Professor Bohlin suggests that the universe has been formed by the breaking down of a great spheroidal shell of matter, of which the equator formed the Galaxy, and the debris of the polar regions the spiral nebulæ.

4. It thus appears that during the last century opinion has been moving steadily in the direction of supposing that the distributions of stars and of nebulæ are more or less mutually exclusive; that this proves that they are not independent of one another, but are complementary parts of one gigantic system; and that any theory of the structure of the universe must explain the process by which the stars and the nebulæ were driven apart or developed apart.

It has seemed to me that the facts are not strong enough to justify such a formal logical treatment; and in a paper communicated to the Cambridge Philosophical Society, based on a lecture delivered before the British Association at Bloemfontein in 1905, I ventured to try the experiment of denying that the distribution of nebulæ is really represented by a concentration on the galactic poles. At that time I had not enough evidence to make the denial convincing, but since then I have been able to collect more material; and in the present note I give a preliminary account of the distribution of spiral nebulæ, so far as it can be deduced from published information.

5. There are two principal difficulties: Surprisingly few nebulæ

have been photographed on an adequate scale ; and our knowledge of different regions of the sky is very unequal.

But the recent publication of Professor Solon Bailey's Catalogue of Bright Clusters and Nebulæ (*Harvard Annals*, **60**, No. 8) has at last provided us with a modest but quite excellent foundation for our proposed structure.

In that catalogue we find all the N.G.C. objects which can be *seen* on plates taken with a one-inch lens with an exposure of one hour. (The Harvard Sky.)

The descriptions are derived from an examination of these objects on plates taken with the Bruce and the Bache telescopes, with longer exposures and on a larger scale.

There are 31 N.G.C. objects described by Bailey as spiral or ? spiral ; and there are 8 more in Bailey's catalogue which are known from published sources to be spiral, though they are not so described by him. Hence we have 39 spirals bright enough to be included in Bailey's Catalogue of 263 objects. I have plotted all these in my figure 1.

6. The projection of the sphere on the circumscribed cube has been used lately by several writers. For our purpose it is convenient to take the galactic plane, instead of the equator, as the plane of symmetry. The central horizontal line of the figure is then the galactic equator, and the galactic poles are in the centres of the top and bottom squares. The projection is far from being an equal area projection ; but it has the advantage that great circles are represented by straight lines which can be traced round the cube with facility. For convenience of studying distributions in the northern and southern hemispheres I have repeated the top and the bottom squares, and one of the central line. The plotted points have not, however, been repeated in this preliminary investigation. The projection is drawn as seen from the outside of the cube. If it is desired to have the inside view, corresponding to the aspect of the celestial sphere, we must interchange the signs of the declinations, and add 12 hours to the R.A.'s.

For ease of reference we will label the squares containing the northern and southern galactic poles **NG** and **SG**. The galactic squares we will call **AG**, **BG**, **CG**, **DG**, and **AG**. The R.A.'s of their centres are about $18^h\ 40^m$, $0^h\ 40^m$, $6^h\ 40^m$, and $12^h\ 40^m$ respectively.

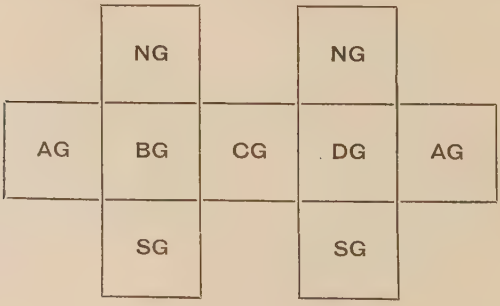


Fig. 1.—Objects in Bailey's Catalogue known to be Spiral Nebulæ.

Fig. 2. Objects in N.G.C. known to be Spiral Nebulæ.

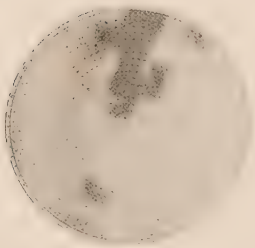


Fig. 3.—Bright Nebulæ in N. Hemisphere (Stratonoff).

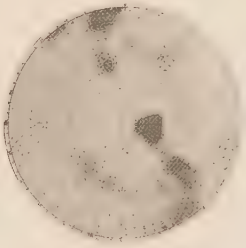


Fig. 4.—Bright Nebulæ in S. Hemisphere (Stratonoff).

It is obvious at a glance that our first batch of 39 spirals are not concentrated about the galactic poles. There is concentration in NG, but nothing corresponding to it in SG; while there are several spirals in BG and DG.

7. The number of objects is, however, very limited. To supplement it I have superposed, in figure 2, all N.G.C. nebulæ whose spiral character is assured, or reasonably certain. The sources are—

(*a*) The published collections of Isaac Roberts and of Keeler, supplemented by descriptions of some unpublished Crossley Reflector plates kindly supplied by Professor Campbell and Dr. Curtis.

(*b*) The drawings and descriptions by Sir John Herschel, Lord Rosse, Lassell, and the Melbourne Observatory.

(*c*) The various catalogues by Max Wolf (*Publ. Koenigstuhl-Heidelberg*, vols. i.–iii.), and published descriptions by Isaac Roberts and others.

(*d*) The lists of nebulæ examined by Frost and Stewart on Bruce plates taken at Harvard or Arequipa. (*Harvard Annals*, **60**, No. 6.)

The second diagram figures the first. We have great numbers in square NG; considerable numbers in SG, BG, and DG; a scattering in CG; and none in AG, except one on the edge of the square, whose spiral character is very uncertain.

It appears, therefore, that the brighter spirals are not disposed symmetrically in galactic latitude. The great patch of spirals in NG has no counterpart in SG. Nor are they symmetrically disposed in galactic longitude. They avoid almost entirely the square AG, the region of Sagittarius, Ophiuchus, and Scorpio, where the Milky Way is bright and filled with immense gaseous nebulæ, and in general they avoid the actual Milky Way zone. But they do it without imposing upon themselves symmetry in either galactic latitude or galactic longitude, and these results suggest that it will be prudent for the future to discuss spiral nebula distribution more on its own merits, and less with an eye to the galactic poles.

8. One may naturally object to accepting any such conclusion, on the ground that long exposure plates taken with powerful reflectors show great numbers of hitherto uncatalogued nebulæ. It has been estimated, for example, that half a million nebulæ are within reach of the Crossley Reflector on Mount Hamilton. How far is it legitimate to draw any conclusion from a couple of hundred nebulæ, while so many remain unexamined?

An examination of the table of new nebulæ found on Crossley Reflector plates is very reassuring. It appears that new spirals are exceedingly plentiful where the old well-known ones are plentiful; but that they do not appear in fields where known spirals are not. If we may assume that all the Crossley Reflector plates have been searched for new nebulæ with equal care, and that on the average the exposures given to those plates do not

differ greatly in different quarters of the sky, we arrive at the following results:—

Square.	Number of Fields.	New Nebulæ per Field.
NG	35	14·9
AG	14	0·0
BG	21	5·4
CG	18	2·0
DG	1	6
SG	11	5·4

These figures, being based upon a reconstruction from data not fully published, cannot be exact; but they serve sufficiently well to show that the proportion of new nebulæ in different squares follows pretty closely the proportion of the old.

A similar examination of the figures given by Messrs. Stewart and Frost leads to the same result.

9. The most obvious deficiency in the data used above is the absence of evidence as to the nature of the nebulæ in the greater cloud of Magellan, situated in the lower left-hand corner of square DG. None of the Harvard plates examined by Stewart and Frost cover this region, and in fact I can find no photographic information as to the presence or absence of spirals in the cloud, yet the point is crucial. Figures 3 and 4 are reproduced from Stratonoff's Atlas (*Publ. Observ. Tachkent*, vol. ii.) to show the distribution of all the nebulæ described in the N.G.C. as bright. The principal condensations shown on figure 2 are easily found in these figures, and the greater Magellanic cloud is most conspicuous. It is equally conspicuous in the charts showing loose clusters, type V stars, or other characteristic Milky Way objects; and many writers have taken the view that galactic and extra-galactic objects, separated elsewhere, come together in the cloud; to which supposed fact a profound significance is not unnaturally attached.

10. It appears to the writer that this evidence is extremely insecure. Stratonoff's charts show essentially the distribution of spirals, because spirals appear to predominate enormously among the nebulæ—the gaseous are relatively so few in number that they make little show. But, on the other hand, it may very well be that in the greater cloud of Magellan the proportion between spiral and gaseous nebulæ is locally inverted—they may be *all* bright gaseous nebulæ, in which case Stratonoff's distribution charts would give a quite false impression, and the argument at the end of the last paragraph would be groundless.

It appears, then, that the classification of the Magellanic nebulæ is greatly to be desired. No spiral is known to be among them. If none are spirals, then a supposed highly significant and at present inexplicable fact is found to be no fact at all, and the ground is wonderfully cleared.

11. These are the considerations which have led me to propose a comparison of the N.G.C. with Mr Franklin-Adams' plates. I am happy to say that, with the most friendly co-operation of the Astronomer Royal, Mr. Hardcastle has volunteered to carry out this examination. A preliminary trial made by us at Greenwich upon a single plate showed that the two or three known spirals were quite conspicuous, and that seven or eight other nebulæ could be classed as spiral with certainty.

12. In conclusion, I must express my thanks to Professor Campbell and Dr. Heber Curtis for their kindness in sending me information from unpublished Crossley plates; to Mr. Franklin-Adams for permission to use his plates in this way; to the Astronomer Royal for his ready help; and to Mr. Hardcastle for his keenness in following up a suggestion easily made, but with laborious consequences. Finally, I desire to thank my pupils Messrs. J. Jackson and J. H. Clarke of Trinity and Mr Vachha of Christ's for assistance in this and in kindred work now under way.

13. In the following tables I give the N.G.C. numbers of all the objects plotted as Spirals in figures (1) and (2).

It will be understood that in many cases the evidence of spiral character is incomplete. I have left out a number of objects which are probably spiral, but of which the published evidence is inconclusive; and have included some for which the evidence cannot be called conclusive, but which from likeness to undoubted spirals may reasonably be classed with them in a preliminary investigation.

TABLE I.

N.G.C. numbers of objects described by Bailey as Spiral or ? spiral.

A. Objects shown spiral on published photographs by Roberts, Keeler, and others:—

224	4254	4501	5055
598	4258	4594	5194
2903	4303	4651	5236
3031	4321	4736	5866
4051	4449	4826	7331
4214	4485, 90	5005	

B. Objects of which there are no published photographs:—

4374	4450		
4382	4472	4526	Total in A and B 31.
4406	4486	4569	

TABLE II.

N.G.C. numbers in Bailey's Catalogue which are known to be Spiral, though not so described by him.

55	253	3034	4656
221	2403	4631	5128

Total 8

TABLE III.

N.G.C. numbers of objects not in Bailey, which are known to be Spiral from the work of Isaac Roberts and of Keeler, and of which in most cases published photographs are available.

205	2841	3631	5457
246	2976	3726	5857
247	3003	4038	6217
613	3079	4088	6412
628	3115	4244	6946
650*	3169	4485	6951
891	3184	4536	7217
908	3198	4559	7448
936	3226, 7	4565	7479
1023	3423	4712	7537
1068*	3556	4725	7782
1084	3623	4747	7814
2371, 2	3627	5247	7817
2683	3628		

Total 54

TABLE IV.

N.G.C. numbers of objects described as Spiral by Max Wolf and Roberts, but of which no published photographs are available.

1659	3731	3995	4535
1665	3878	4020	4635
1667	3935	4062	4685
3593	3971	4136	4774
3596	3986	4150	4846
3666	3991	4448	4868
3692	3994	4455	4914
3705			

Total 29

* 650 and 1068 have all the appearance of spirals on the Crossley plates 5 and 7; but their spectra show bright lines, and their character is therefore dubious.

TABLE V.

N.G.C. numbers of objects not described as Spirals from photographs, but which from published drawings and descriptions are probably Spirals.

134	1808	4346	5394
676	1888	4389	5427
693	3109	4414	5544
808	3395	4567	5545
833	3521	4568	5907
839	3893	4618	7177
986	3938	4762	7457
1012	3953	4861	7678
1514	4123	5033	7769
1531	4192	5112	7771
1637	4216	5248	

Total 42

TABLE VI.

N.G.C. numbers of objects described as Spiral by Stewart and Frost from photographs taken at Arequipa.

150	4178	4438	6221
1187	4189	4440	6300
1249	4206	4498	6699
1288	4208	4548	6719
1365	4294	4579	6744
1433	4298	4639	6862
1559	4299	4654	6872
1566	4388	4710	7083
1672	4394	5182	7125
2835	4402	5483	7329
2997	4413	5643	7734
3621	4429	6215	

Total 47

Cambridge Observatory:
1911 *May* 12.

On the Harvard Eclipses of Jupiter's Satellite IV.

By W. de Sitter, Assoc. R.A.S.

The accompanying table contains the result of a comparison of the eclipses of Satellite IV. with my theory by means of the formulæ given in my paper in *M.N.*, lxxi. p. 85. The observations, as is well known, consist for each phenomenon of a series of photometric determinations of the brightness of the satellite at stated times. From these the instant of half light, which is the datum to be used for the comparison with the theory, has been derived by two independent and entirely different methods at Harvard and at Durham. At Harvard five observations are selected from which the instant of half light is derived, with the aid of an adopted light-curve, the same for all cases. Professor Sampson also uses a theoretical light-curve, which is, however, varied according to the circumstances of the eclipse, and compares it with the whole of the observations. I have adopted the Durham readings.

The rotation-numbers given in the first column are those of Professor Sampson's table on p. 197 of *Harvard Annals*, vol. lii., pt. ii. The observed Greenwich mean time is derived as follows:—

$$\text{G.M.T.} = \text{pred}^{\text{d}}.\ \text{time of Am. Eph.} + \text{col. } \mathbf{11} - \text{col. } \mathbf{14} + 5^{\text{h}}\ 8^{\text{m}} \begin{cases} 12^{\text{s}} \\ 16 \end{cases}.$$

The quantity given in column **14** is the correction I. explained on page 201, with the sign reversed. For the one or two cases where it is not contained in the table, I have computed it. The corrections II.A and II.B (pages 201 and 202), which should also have been applied, are negligible. The difference of longitude between Washington and Greenwich adopted in the American Ephemeris was $5^{\text{h}}\ 8^{\text{m}}\ 12^{\text{s}}$ up to 1899, and $5^{\text{h}}\ 8^{\text{m}}\ 16^{\text{s}}$ after that year. The observed times thus derived are all that is borrowed from Professor Sampson's memoir. The comparison with the theory is then carried out, as has already been said, by the formulæ given in my paper of last November. The adopted values of the elements are those of my theory of 1908, and the values of the diameter and compression of Jupiter are the same as stated in *M.N.*, lxxi. p. 96.

The meaning of v, r, β, C, c, y, z' is explained in the paper quoted; n_{I} is derived as explained there (p. 97):—

$$2n_{\text{I}} = \frac{y^2 + z'^2(1+c)^2}{\text{C}} - \text{C},$$

The mean longitude used in my theory is

$$l_4 = 234^\circ\cdot372 + 21^\circ\cdot57110965\ t,$$

the time being counted from 1900 Jan. 0·0 G.M.T.

The value given in Professor Sampson's tables is

$$234^\circ\cdot40790 + 21^\circ\cdot571109630\, t.$$

The difference undoubtedly must be explained by a difference in the definition of the term "mean longitude." My longitude is longitude in the orbit, thus—

$$\Upsilon N + N\Omega + \Omega S,$$

where Υ is the mean equinox, N is the node of the fundamental plane on the ecliptic, Ω the node of the satellite's instantaneous orbit on the fundamental plane, and S the position of the satellite.

The residual n can be expressed in time, by assuming the position of the orbit as given by the theory to be correct. We have then with sufficient accuracy

$$O - C = \mp n \sec\gamma, \begin{cases} -in \\ +ex \end{cases}$$

where

$$\tan\gamma = \frac{z'(1+c)}{y}.$$

In this manner were derived the values given under O – C, I. It will be seen that these are to a small extent systematically negative. This points to a positive correction to the mean longitude. Omitting the observation No. 18, and treating the others as of equal weight, I find

$$\delta l = +0^\circ\cdot0047.$$

My adopted longitude was the mean of three independent determinations, which stand thus:—

		Corrections to Damoiseau.
Gill—de Sitter,	1891	$-0^\circ\cdot0248 \pm 0^\circ\cdot0010$
Cookson	1901	$-0\cdot0361 \pm \cdot0018$
Cookson	1902	$-0\cdot0342 \pm \cdot0016$
	Adopted mean	$-0\cdot030$

We now find:

Harvard eclipses $-0^\circ\cdot0253$,

or exactly the same as derived from the heliometer-observations of 1891. This result leads us to reconsider Cookson's two determinations. Their deviation from the value of 1891 is too large for the probable error, but at the time when I had to decide on a mean value there was no reason to give preference to any one of the three results. Now that the result of 1891 is confirmed by the

Harvard eclipses, it is hardly possible to escape the conclusion that Cookson's two values must be affected by some systematic error. I am, however, not able to point out the origin of this error. The only thing that I can suggest is the fact that Cookson neglected to apply the correction for the motion of the system during the interval of time needed for the light to reach the satellite after passing the planet's edge, which has been explained in art. 13 of my paper of November last.* This can, however, hardly have had a systematic effect, and can certainly not exceed a few units of the fourth decimal place in the mean of a prolonged series of extra-eclipse observations.

I propose thus to adopt for the correction to Damoiseau

$$-0^\circ\cdot0250.$$

The correction to my theory is then

$$+0^\circ\cdot0050,$$

and the longitude for 1900 Jan. 0·0 becomes

$$234^\circ\cdot3770,$$

The uncertainty of this value can hardly be more than $\pm 0^\circ\cdot0010$.

Introducing this value, I find the residuals n_{II}, and transforming these into time we find the values O – C, II.

Professor Sampson has in *M.N.*, lxxi. pages 80 and 81, given a comparison of these same observations with his tables, only omitting those with weights 1 and 2. He adopts the mean of the Harvard and Durham readings. To make our (O – C)'s comparable I have added to the values II. the difference between the observed times as stated by Professor Sampson and those used in this paper. The result is the column O – C, III. Professor Sampson's residuals are added for comparison.

It will be seen that on the whole the values II. are appreciably smaller than III. The Durham readings thus appear to be much better than the Harvard readings. There are a few exceptions, the only important one being No. 27. The difference between the Durham and Harvard readings for this observation was more than ten minutes. It is one of those quoted by Professor Sampson as

* So far as I can see, this correction has also been overlooked by Professor Sampson, as well as by Damoiseau and other investigators. So long as eclipses alone are considered, the correction is constant, and its neglect is therefore unimportant, and simply amounts to a difference in the definition of longitude, the conjunction being taken with reference to a ray of light, instead of to a geometrical (time-less) straight line. But when comparing longitudes derived from eclipses with those derived from observations taken away from conjunction, this difference of definition should be remembered. In a general theory, applicable to all cases, it seems preferable to adopt the geometrical definition.

having a modified light-curve owing to high latitude [*Harvard Annals*, lii. **2**, plate iv., fig. 4]. In a definitive discussion it would perhaps be better to exclude this observation.

The residuals O – C for observations in high latitudes (large values of z') are on the whole somewhat larger than for the others, or, to be more precise, the large (O – C)'s as a rule belong to observations in high latitudes. That this is due to the unavoidable uncertainty of the observations of these slow eclipses, and not to the theory, is at once apparent from the residuals n.

The observation No. 18 is evidently erroneous. It was rejected by the author of the Harvard readings.

The present paper is not intended as a comparison between Professor Sampson's results and mine. Such a comparison is, of course, highly desirable, but it can hardly be profitably carried out until Professor Sampson's theory and the data on which his tables are based have been completely published. Thus, *e.g.*, the four numbers which Professor Sampson quoted for the proper inclinations of the satellites in the *Observatory* (January 1911, page 55) occur nowhere in his published writings; at least I have not been able to find them either in the *Harvard Annals* or in the Tables.

I have also not tried to derive corrections to my theory beyond the correction to the mean longitude mentioned above. Undoubtedly the residuals could be improved — though I do not think the improvement would be very considerable—by small corrections to the excentricity and perijove, and the other elements. Such corrections would, of course, have to be combined with their proper weights with those derived from other sources [among which the most important are those resulting from the Cape observations, which, of course, are zero]. For the derivation of a definitive theory we shall, however, have to await the completion of the series of observations which are now being made, such as those of Mr. Innes, and the photographs which are being made at Pulkovo and Simeïs, and also the new determination of the inclinations and nodes by heliometric or photographic measures at about the epoch 1920, of which I have pointed out the desirability. (See *Cape Annals*, xii. **3**, p. 121; *Proceedings Amsterdam Academy*, 1908, March, page 720.) In the meantime I think my theory is amply sufficient to serve as a basis for the reduction of the observations, and its utility for that purpose would not be appreciably enhanced by such corrections as can now be derived.

[Table.

No.	Date.	G.M.T. d	h	m	s	*v.* °	*r.*	*β.* c	C.	Log $(1+e)$.
1 *ex*	1878 Sept.	1	15	19	52	307·6502	0·99435	−0·1607	2·1792	0·02799
2, 3 *in*	Nov.	7	11	20	55	309·2278	0·99453	−0·1717	92	801
4 *in*	1879 Aug.	2	14	6	36	333·1073	0·99677	−0·3128	93	795
5, 6 *ex*			18	37	32	337·1896	0·99724	−0·3306	93	795
7 *in*	Oct.	8	15	5	35	339·2764	0·99761	−0·3400	93	787
8 *ex*		25	13	30	57	344·5581	0·99820	−0·3607	93	786
9 *in*	1883 Sept.	18	21	42	0	107·9325	1·00717	+0·0212	91	786
10 *ex*	Oct.	5	19	18	48	112·4586	1·00685	+0·0534	91	788
11 *in*	Nov.	24	21	39	51	113·2086	1·00680	+0·0590	91	793
12 *ex*	Dec.	28	13	51	42	119·5802	1·00662	+0·1037	90	796
13 *ex*	1884 Mar.	4	14	18	18	125·1454	1·00608	+0·1420	90	797
14 *in*	Apr.	23	15	56	35	125·0820	1·00609	+0 1418	90	799
15 *ex*	May	10	14	43	44	130·6052	1·00566	+0·1780	90	800
17 *in*	1886 Feb.	22	16	25	16	177·9045	1·00065	+0·3914	89	767
18 *in*	Apr.	30	16	56	41	183·6257	0·99996	+0·4013	89	760
19 *ex*			18	11	56	184·7536	0·99981	+0·4027	89	760
20 *in*	1889 Aug.	18	14	25	58	277·0311	0·99262	+0·0509	91	775
21 *in*	1890 July	19	14	57	22	304·2468	0·99355	−0·1392	92	799
22 *ex*	Aug.	5	13	42	38	309·9115	0·99385	−0·1744	92	800
23 *in*	Sept.	24	15	28	24	309·9984	0·99385	−0·1754	92	801
24 *ex*	1891 July	6	17	4	54	339·4801	0·99659	−0·3297	93	791
25 *ex*	Sept.	28	11	51	32	346·8483	0·99762	−0·3576	93	783
26 *in*	1892 Feb.	9	10	34	28	356·0356	0·99859	−0·3803	93	763
27 *in*	1895 Feb.	19	13	22	14	95·5257	1·00753	−0·0559	91	791
28 *in*	Oct.	28	19	43	3	115·2340	1·00721	+0·0788	90	795
29 *in*	1896 Jan.	3	19	41	28	120·5140	1·00684	+0·1140	90	797
30 *in*	Mar.	27	13	51	34	127·0927	1·00664	+0·1564	90	800
31 *ex*	Apr.	13	12	41	24	132·6495	1·00613	+0·1900	90	801
32 *in*	June	2	14	4	30	132·3625	1·00615	+0·1888	90	802
33 *ex*		19	12	59	0	138·0022	1·00586	+0·2213	90	803
34 *in*	Dec.	3	20	8	24	146·8228	1·00508	+0·2685	89	797
35 *ex*		20	18	51	27	152·3621	1·00443	+0·2942	89	797
36 *ex*	1897 Feb.	25	18	36	27	157·4360	1·00408	+0·3157	89	794
37 *in*	Nov.	3	20	33	57	173·2438	1·00231	+0·3659	89	774
38 *in*	1898 Jan.	9	20	35	7	178·6254	1·00153	+0·3766	89	767
39 *ex*			22	53	24	180·6917	1·00126	+0·3797	89	767
40 *ex*	1901 July	23	14	50	50	281·1416	0·99260	+0·0099	91	777
41 *in*	Sept.	28	12	33	2	284·2854	0·99256	−0·0116	91	782
42 *in*	1902 June	6	19	17	15	305·0450	0·99308	−0·1462	92	799
43 *ex*		23	18	0	45	310·7064	0·99352	−0·1797	92	800
44 *ex*	Sept.	15	12	48	0	318·0898	0·99387	−0·2211	93	803
45 *in*	1903 July	30	17	54	37	342·5266	0·99612	−0·3279	93	784
46 *in*	Oct.	22	13	19	42	350·4306	0·99702	−0·3501	93	777
47 *ex*			16	41	25	353·4793	0·99739	−0·3568	93	777

No.	y.	z'.	$n_{I.}$	$n_{II.}$	O − C. I.	O − C. II.	O − C. III.	O − C. Sampson.	Wt.
					s	s	s	s	
1	+2·0573	−0·6934	+0·0070	+0·0117	+ 30	+ 50	+ 65	+150	3
2, 3	−2·1394	−0·3923	+ ·0007	− ·0042	− 3	+ 17	+142	+ 97	3, 3
4	−2·0111	+0·7328	− ·0215	− ·0261	+ 93	+ 112	...	...	1
5, 6	+2·0394	+0·7159	− ·0017	+ ·0030	− 7	+ 13	+ 26	− 32	4, 1
7	−1·8873	+1·0088	− ·0070	− ·0113	+ 32	+ 52	...	...	2
8	+1·8546	+1·0634	− ·0055	− ·0013	− 26	− 6	+ 7	− 97	3
9	−1·5744	+1·4030	− ·0073	− ·0109	+ 41	+ 60	+ 48	+ 26	5
10	+1·5821	+1·3630	− ·0311	− ·0275	− 170	− 150	−164	− 60	5
11	−1·7949	+1·1638	+ ·0030	− ·0011	− 15	+ 5	+ 22	+ 48	5
12	+1·8574	+1·0655	− ·0016	+ ·0027	− 7	+ 13	− 15	+ 56	4
13	+2·0020	+0·8178	+ ·0046	+ ·0092	+ 20	+ 40	+ 17	+ 67	5
14	−2·1023	+0·6013	+ ·0190	+ ·0142	− 80	− 60	− 39	+ 3	5
15	+2·0904	+0·5641	− ·0037	+ ·0011	− 15	+ 5	− 6	+ 38	5
17	−1·1133	−1·7617	+ ·0039	+ ·0014	− 31	− 11	+ 6	−133	5
18	−0·4572	−1·9352	− ·0656	− ·0667	+1220	+1241	...	...	1
19	+0·6667	−1·9336	− ·0133	− ·0118	− 174	− 155	−197	+405	4
20	−0·9221	−1·8402	− ·0114	− ·0135	+ 108	+ 128	+175	−165	5
21	−2·0689	−0·6409	− ·0003	− ·0050	+ 1	+ 21	+ 29	− 46	4
22	+2·0985	−0·5972	+ ·0139	+ ·0187	+ 58	+ 79	+ 59	+101	5
23	−2·1482	−0·3635	+ ·0037	− ·0012	− 15	+ 5	+ 13	− 48	5
24	+1·9949	+0·8266	+ ·0017	+ ·0063	+ 7	+ 28	+ 45	− 78	5
25	+1·7786	+1·1708	− ·0063	− ·0022	− 31	− 11	+ 6	−122	5
26	−1·2301	+1·6890	+ ·0008	− ·0020	− 6	+ 14	...	...	2
27	−0·7438	+1·9409	+ ·0211	+ ·0194	− 251	− 231	+ 80	−348	4
28	−1·8501	+1·0870	+ ·0042	·0000	− 20	0	+ 27	+ 30	5
29	−1·9943	+0·8360	+ ·0055	+ ·0009	− 24	− 4	+ 14	+ 30	5
30	−2·1273	+0·5152	+ ·0182	+ ·0133	− 76	− 55	− 45	0	5
31	+2·1062	+0·4743	− ·0128	− ·0080	− 53	− 33	− 44	− 5	4
32	−2·1750	+0·2565	+ ·0132	+ ·0082	− 53	− 33	− 18	+ 32	5
33	+2·1505	+0·2154	− ·0162	− ·0113	− 65	− 46	− 64	− 16	5
34	−2·1350	−0·4475	+ ·0088	+ ·0039	− 36	− 16	+ 19	+ 58	5
35	+2·1123	−0·4909	− ·0026	+ ·0022	− 11	+ 9	− 23	+ 72	5
36	+2·0155	−0·7390	− ·0147	− ·0101	− 64	− 44	− 50	+ 65	4
37	−1·3547	−1·5951	− ·0049	− ·0080	+ 32	+ 52	+ 71	− 87	4
38	−1·0474	−1·7907	− ·0018	− ·0042	+ 15	+ 35	+ 96	−189	4
39	+1·0146	−1·7874	− ·0205	− ·0182	− 175	− 156	−242	+175	4
40	+1·0123	−1·7965	− ·0128	− ·0105	− 110	− 90	...	...	1
41	−1·4081	−1·5639	+ ·0033	+ ·0001	− 21	− 1	+ 13	−121	5
42	−2·0728	−0·6135	− ·0055	− ·0102	+ 23	+ 43	+ 29	− 17	3
43	+2·0886	−0·5683	− ·0044	+ ·0004	− 18	+ 2	+ 20	+ 58	5
44	+2·1532	−0·2140	− ·0140	− ·0091	− 57	− 37	− 27	− 16	5
45	−1·8120	+1·1573	+ ·0134	+ ·0093	− 65	− 45	− 52	+ 39	3
46	−1·5256	+1·4848	+ ·0191	+ ·0156	− 111	− 90	−106	+ 30	5
47	+1·5016	+1·4790	− ·0025	+ ·0009	− 15	+ 5	+ 27	− 87	5

Note on Professor de Sitter's Paper.

By R. A. Sampson, D.Sc., F.R.S.

Damoiseau's equation of light, to which reference is made in the last paper, duly includes a term arising from the difference of the distances of the satellite and Jupiter from the Earth. It is derived from Delambre, who gives the theory in the *Introduction* to his tables, p. vii:—"L'équation que l'on vient de voir n'est exacte que pour Jupiter; mais les satellites, au moment de la conjonction, sont plus éloignés du Soleil d'une quantité égale à leur rayon vecteur," etc.

Damoiseau does not expressly mention it, but a verification that he included the values which Delambre derived in the constant part of his equation may be found in *Harvard Annals*, vol. 52, part 2, p. 211.

Note on Professor Sampson's Note.

By W. de Sitter, Assoc. R.A.S.

The term mentioned by Professor Sampson is not the one referred to in the footnote to my paper. As the two corrections are easily confounded, I may be allowed to explain at somewhat greater length.

We treat the Sun and the satellite as points, and suppose all bodies concerned to be in one and the same plane. We have to consider two rays of light, viz.—

(a) the outgoing ray from the Sun to the satellite, which grazes Jupiter's rim on the way; and

(b) the returning ray from the satellite to the Earth.

The distance traversed by the latter is $\Delta + r\cos(v - \mathrm{L})$, and the second term gives rise to the correction to the equation of light quoted by Professor Sampson. It has, so far as I know, been correctly applied by all investigators. It is a correction to the *observed* time. It depends on the annual parallax of Jupiter (*i.e.* on the position of the Earth), but is in practice taken as constant, the adopted mean value being approximately—

$$\delta t_0 = ka = \tau,$$

k being the constant of aberration, and a the satellite's mean distance.

The effect which I referred to in my paper results from the consideration of the outgoing ray a: it is independent of the position of the Earth, and it affects the *true* time of the phenomenon. This ray a, after passing the edge of the planet, continues its way along the same straight line in space on which the phenomena take place. But this line differs from that on which the conjunction takes place, which is carried along with Jupiter in

its motion round the Sun. (We are here, of course, speaking of conjunctions of the satellite with the rim, and not with the centre, of the planet.) Consequently the phenomenon takes place before the satellite reaches the point on its orbit for which the conditions of geometrical conjunction are satisfied. The (geometrically) computed time t thus requires a negative correction.

Taking another point of view, let us imagine an observer on the satellite—say the fourth satellite, Callisto. He sees an eclipse of the Sun at the instant t at which the apparent longitudes of the Sun and of Jupiter's rim are equal. We, terrestrial astronomers, prefer not to use the "Callistonian" apparent longitudes, but true longitudes. The true longitude at time $t - \Delta t$ is equal to the apparent longitude at time t, but the equation of light Δt is different for the Sun and for Jupiter. Therefore the true time t of the eclipse, which is the moment of equal apparent (Callistonian) longitudes, differs from the time of equal true longitudes, or geometrical conjunction.

The correction to be applied to the time of true conjunction is

$$\delta t = -\frac{\mathrm{R}+a}{a}\,\frac{\mathrm{N}}{n-\mathrm{N}}\cdot\tau,$$

where τ is the same as above. It so happens that the factor differs little from unity, and the two corrections nearly cancel each other for the three inner satellites. The corrections to the observed time are (see, *e.g.*, *Harvard Annals*, lii. 2, page 211)—

$$\delta t_0 = +1^{\mathrm{s}}\cdot4, \quad +2^{\mathrm{s}}\cdot2, \quad +3^{\mathrm{s}}\cdot5, \quad +6^{\mathrm{s}}\cdot2.$$

The correction to the true time of conjunction is (*M.N.*, lxxi. page 92) $0^{\circ}\cdot0025$ in jovicentric arc, corresponding for the four satellites to—

$$\delta t = -1^{\mathrm{s}}\cdot0, \quad -2^{\mathrm{s}}\cdot1, \quad -4^{\mathrm{s}}\cdot3, \quad -10^{\mathrm{s}}\cdot0.$$

For shadow-transits δt_0 changes its sign, but δt remains negative: here the two corrections are in the same sense. For occultations and transits the correction δt_0 is practically the same as for eclipses and shadow-transits respectively: δt, however, now depends on the relative position of Jupiter and the Earth, and can no longer be assumed constant (see *M.N.*, lxxi. page 93).

Erratum in Dr. de Sitter's paper on the Principle of Relativity, in Monthly Notices, vol. lxxi.

p. 391, line 6 from bottom, in the denominators, *for* (κ) *read* $\sqrt{-1\phi^2}$.